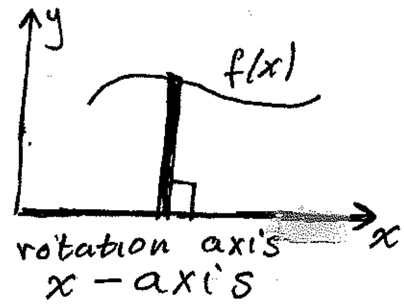
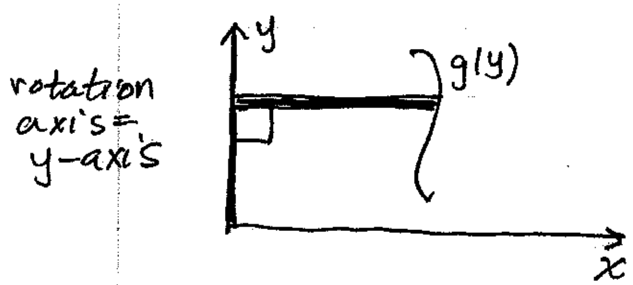
Disks/washers



radius $= f(x)$

$\pi \int_a^b (f(x))^2\, dx$

If $y=a$ is the axis, $r = |f(x)-a|$



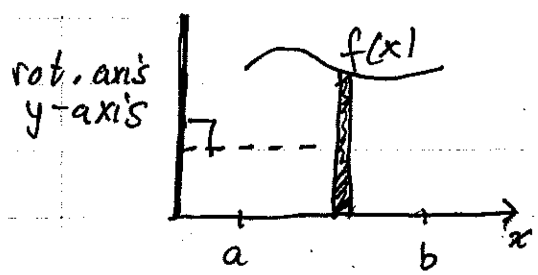
radius $= g(y)$

$\pi \int_c^d (g(y))^2\, dy$

If $x=a$ is the axis,

radius $= |g(y)-a|$

Cylindrical Shells



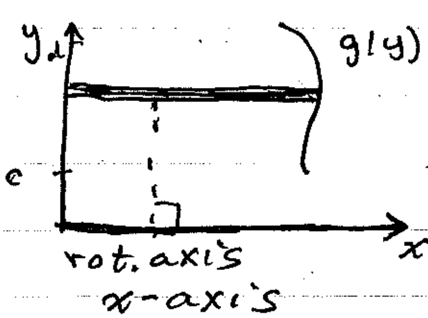
radius $= x$

height $= f(x)$

$2\pi \int_a^b x\, f(x)\; dx$

If axis is $x=a$:

$r = |x-a|$



radius $= y$

height $= g(y)$

$2\pi \int_c^d y\, g(y)\, dy$

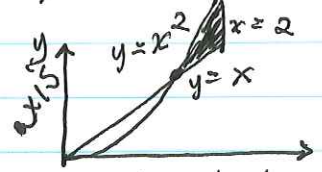
If axis $y=a$:

$r = |y-a|$

7.3 Solutions to Notes Problems J. Lewis

Cylindrical Shells and Washers

1.



The shaded region is rotated about the y-axis.

Shells

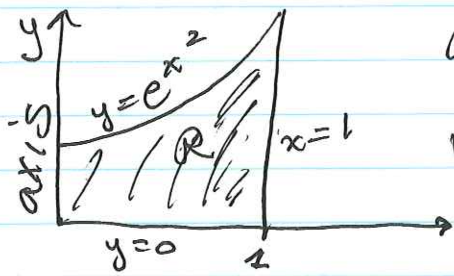
$$V = 2\pi \int_1^2 x(x^2 - x)\,dx$$

$$= 2\pi \int_1^2 x^3 - x^2\,dx = 2\pi\left(\tfrac{1}{4}x^4 - \tfrac{1}{3}x^3 \Big|_1^2\right)$$

$$= 2\pi\left(4 - \tfrac{8}{3} - \left(\tfrac{1}{4} - \tfrac{1}{3}\right)\right) = 2\pi\left(4 - \tfrac{7}{3} - \tfrac{1}{4}\right)$$

$$= \boxed{\tfrac{17}{6}\pi}$$

2.



R is rotated about the y-axis.

$$V = 2\pi \int_0^1 x e^{x^2}\,dx$$

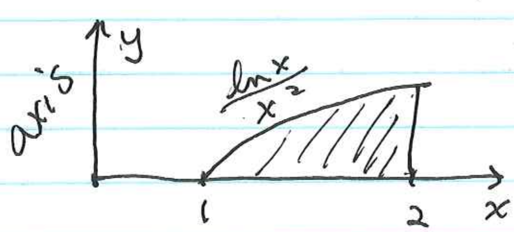
$u = x^2$ $u(0) = 0$ $u(1) = 1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$V = \cancel{2}\pi \int_0^1 \tfrac{1}{\cancel{2}} e^u\,du = \pi e^u \Big|_0^1 = \boxed{\pi(e-1)}$$

3.



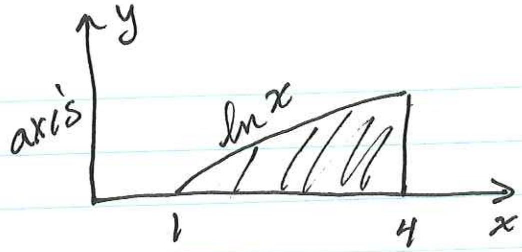
$$V = 2\pi \int_1^2 x \frac{\ln x}{x^2}\,dx$$

$$= 2\pi \int_1^2 \frac{\ln x}{x}\,dx$$

$u = \ln x$ $u(1) = 0$ $u(2) = \ln 2$

$du = \frac{1}{x}dx$ $V = 2\pi \int_0^{\ln 2} u\,du = \cancel{2}\pi\left(\tfrac{1}{\cancel{2}}(\ln 2)^2\right)$

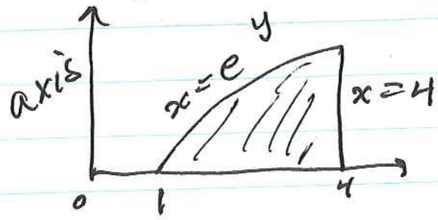
4.



$$V = 2\pi \int_1^4 x \ln x \, dx$$ using shells.

Using washers:

$R = 4 \quad r = e^y \quad 0 \le y \le \ln 4$



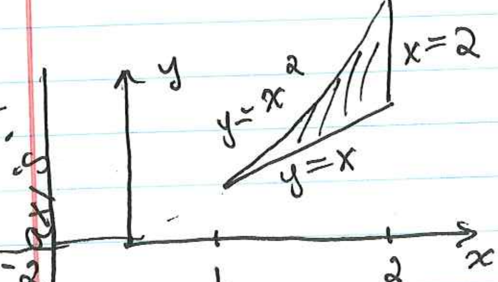
$$\pi \int_0^{\ln 4} 4^2 - (e^y)^2 \, dy$$

$$V = \pi \int_0^4 16 - e^{2y} \, dy = \pi\left(16y - \tfrac{1}{2}e^{2y} \Big|_0^{\ln 4}\right)$$

$$= \pi\left(16 \ln 4 - 8 - \left(0 - \tfrac{1}{2}\right)\right)$$

$$= \pi(16 \ln 4 - 7.5)$$

5.



R is rotated about $x = -2$. Use shells

$r(x) = x+2 \quad h(x) = x^2 - x$

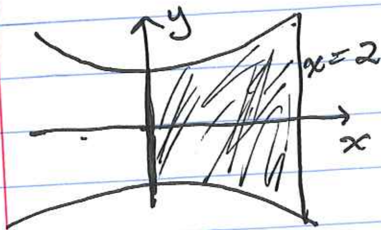
$$V = 2\pi \int_1^2 (x+2)(x^2 - x) \, dx$$

$$= 2\pi \int_1^2 x^3 + x^2 - 2x \, dx = 2\pi\left(\tfrac{1}{4}x^4 + \tfrac{1}{3}x^3 - x^2 \Big|_1^2\right)$$

$$= 2\pi\left(\cancel{4} + 8/3 - \cancel{4} - \left(\tfrac{1}{4} + \tfrac{1}{3} - 1\right)\right)$$

$$= 2\pi\left(\tfrac{37}{12}\right) = \boxed{\tfrac{37}{6}\pi}$$

6.



$y^2 - x^2 = 1$

a) R is rotated about the y-axis.
Use shells.

$$V = 2\pi \cdot 2 \cdot \int_0^2 x\sqrt{1+x^2}\,dx$$

double the top half

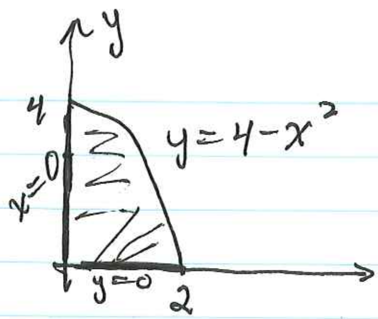
$u = 1+x^2$
$du = 2x\,dx$

$$V = 2\pi \int_1^5 \sqrt{u}\,du$$
$$= 2\pi \cdot \frac{2}{3} u^{3/2} \Big|_1^5$$
$$= \boxed{\frac{4\pi}{3}(5\sqrt{5} - 1)}$$

b) The top half is rotated about the x-axis.
Use washers.

$$V = \pi \int_0^2 (\sqrt{1+x^2})^2\,dx = \pi \int_0^2 1+x^2\,dx$$
$$= \pi \left[x + \frac{1}{3}x^3 \Big|_0^2 \right] = \pi \left[2 + \frac{8}{3} - 0 \right]$$
$$= \frac{14}{3}\pi$$

7.



$\mathbb{R}$ is rotated

a) about the y-axis

Use shells

$$2\pi \int_0^2 x(4-x^2)\,dx = 2\pi \int_0^2 4x - x^3\,dx$$

$$= 2\pi\left[2x^2 - \tfrac{1}{4}x^4 \Big|_0^2\right] = 2\pi\left[8 - 4 - 0\right]$$

$$= \boxed{8\pi}$$

b) $\mathbb{R}$ is rotated about the x-axis.

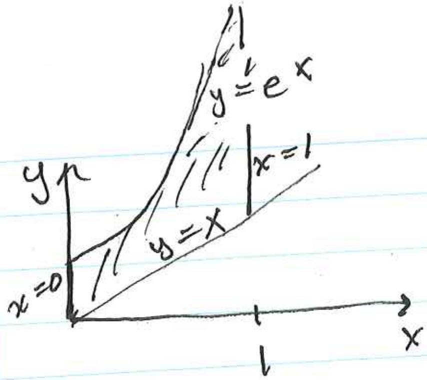
Use disks.

$$V = \pi \int_0^2 (4-x^2)^2\,dx = \pi \int_0^2 16 - 8x^2 + x^4\,dx$$

$$= \pi\left[16x - \tfrac{8}{3}x^3 + \tfrac{1}{5}x^4 \Big|_0^2\right] = \pi\left[32 - \tfrac{64}{3} + \tfrac{16}{5}\right]$$

$$= \pi\left(\frac{208}{15}\right)$$

8.



For set-up only.

a) R is rotated about the x-axis.
Washers.

$$V = \pi\int_0^1 (e^x)^2 - x^2\,dx = \pi\int_0^1 e^{2x} - x^2\,dx$$

$$= \pi\left[\frac{1}{2}e^{2x} - \frac{1}{3}x^3\Big|_0^1\right] = \pi\left(\frac{1}{2}e^2 - \frac{1}{3} - \frac{1}{2}\right]$$

$$= \pi\left(\frac{1}{2}e^2 - 5/6\right)$$

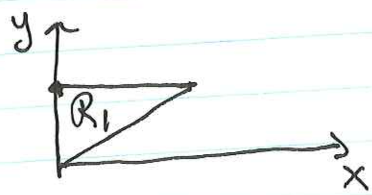
b) R is rotated about the y-axis.
Shells.

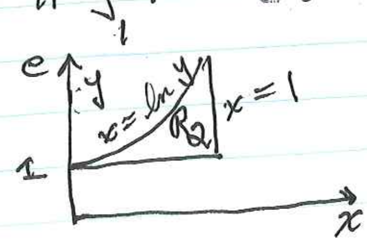
$V = 2\pi\int_0^1 x(e^x - x)\,dx$ set up.

This integral requires integ. by parts.

Washers

$$V = \pi\int_0^1 y^2\,dy + \pi\int_1^e 1 - (\ln y)^2\,dy$$





This 2nd one also uses integ. by parts.