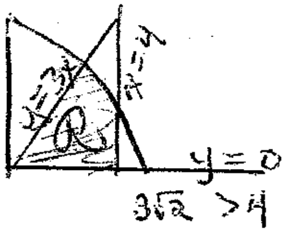
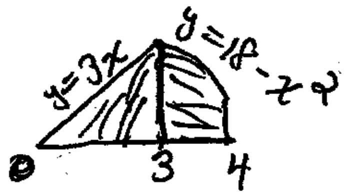
Ex 1 Review 152 Solutions

1 a)





$$18 - x^2 = 3x$$
$$0 = x^2 + 3x - 18$$
$$0 = (x+6)(x-3)$$
$$x = -6 \quad \boxed{x = 3}$$

$$A = \int_0^3 3x \, dx + \int_3^4 18 - x^2 \, dx$$
$$= \frac{3}{2}x^2 \Big|_0^3 + \left(18x - \frac{1}{3}x^3\right)\Big|_3^4$$
$$= \frac{27}{2} + 18 \times 4 - \frac{64}{3} - \left(18 \times 3 - \frac{27}{3}\right)$$
$$= 115/6$$

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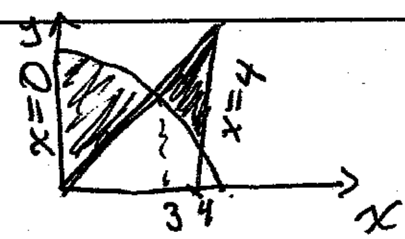
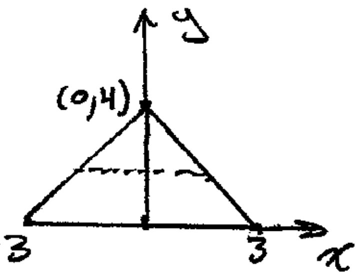
1 b)



$$A = \int_0^3 18 - x^2 - 3x \, dx + \int_3^4 3x - (18 - x^2) \, dx$$
$$= \frac{109}{3}$$

2. a) 

The radius of the semicircle at y is $x = 3 - 3/4\, y$

$\frac{y}{4} + \frac{x}{3} = 1$ is the intercept form of the line.

$\frac{3}{4}y + x = 3$

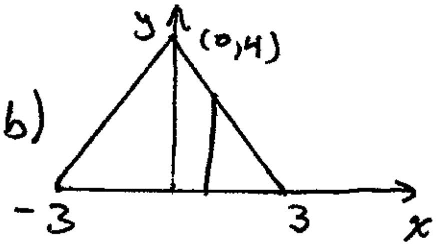
$x = 3 - 3/4\, y$

Or using similar Δ's $\frac{4-y}{x} = \frac{4}{3}$ and solve for.

The area at y is $\frac{\pi}{2}(3 - 3/4\, y)^2$

$$\text{Volume} = \frac{\pi}{2}\int_0^4 (3 - 3/4\, y)^2\,dy = \frac{\pi}{2}\left(-\frac{4}{9}(3 - 3/4\, y)^3 \Big|_0^4\right)$$

$$= \frac{\pi}{2}\cdot\frac{4}{9}\cdot 3^3 = \boxed{6\pi}$$

2. b) 

The <u>diameter</u> of the semicircle at x is $y = 4 - \frac{4}{3}x$.
The area at x is

$$\frac{\pi}{2}\left(2 - 2/3\, x\right)^2$$

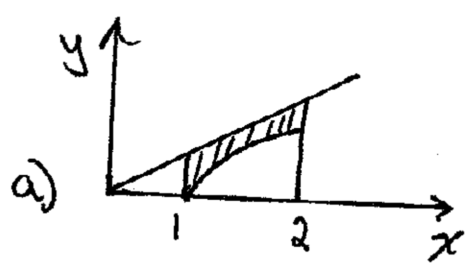
$$\text{Volume} = \int_{-3}^{3} \frac{\pi}{2}\left(2 - 2/3\, x\right)^2 dx = 2\int_0^3 \frac{\pi}{2}\left(2 - \frac{2}{3}x\right)^2$$

$$= \pi \int_0^3 \left(2 - \frac{2}{3}x\right)^2 dx$$

$$= -\frac{3}{2}\pi\frac{1}{3}\left[2 - \frac{2}{3}x\right]^3 \Big|_0^3$$

$$= \frac{\pi}{2}(8) = \boxed{4\pi}$$

3. a)



$$V = \pi\int_1^2 x^2 - (\ln x)^2\,dx = \pi\int_1^2 x^2\,dx - \pi\int_1^2 (\ln x)^2\,d$$

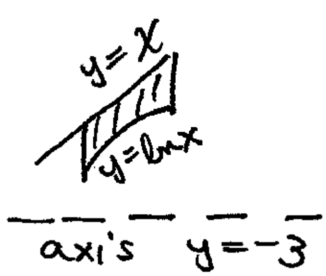
$\int (\ln x)^2\,dx$ is done by parts : $u = (\ln x)^2$ $dv = dx$
$du = \frac{2\ln x}{x}dx$ $v = x$

$$= x(\ln x)^2 - \int 2\ln x\,dx \qquad u = \ln x \quad dv = 2dx$$
$$du = \frac{1}{x}dx \quad v = 2x$$

$$= x(\ln x)^2 - 2x\ln x + 2\int dx$$
$$= x(\ln x)^2 - 2x\ln x + 2x$$

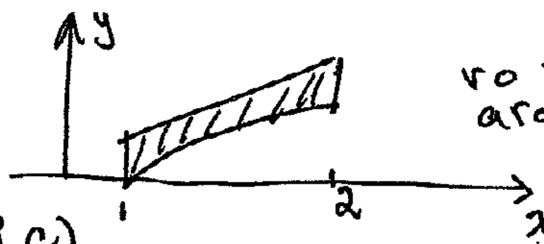
$$V = \pi\frac{1}{3}x^3\Big|_1^2 - \pi\left[x(\ln x)^2 - 2x\ln x + 2x\Big|_1^2\right]$$
$$= \frac{7}{3}\pi \qquad - \pi\left[2(\ln 2)^2 - 4\ln 2 + 2\right]$$

3 b)



$R = x - (-3) = x + 3$
$r = \ln x - (-3) = \ln x + 3$

$$V = \pi\int_1^2 (x+3)^2 - (\ln x + 3)^2\,dx \cong 8.82725$$



rotated around the y-axis:

3 c)

$$V = 2\pi \int_1^2 x(x - \ln x)\,dx = 2\pi \int_1^2 x^2 - x\ln x \; dx$$

$$\int x \ln x \, dx \overset{\text{by parts}}{=} \tfrac{1}{2}x^2 \ln x - \int \tfrac{1}{2} x \, dx = \tfrac{1}{2}x^2 \ln x - \tfrac{1}{4}x^2$$

$$V = 2\pi \left[\tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 \ln x - \tfrac{1}{4}x^2 \Big|_1^2 \right]$$

$$= 2\pi \left[\tfrac{8}{3} - 2\ln 2 - 1 - \left(\tfrac{1}{3} - 0 - 1/4 \right) \right]$$

$$= 2\pi \left[7/3 - 2\ln 2 - 3/4 \right] \approx 1.238$$

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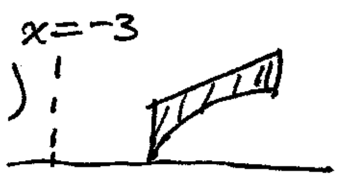
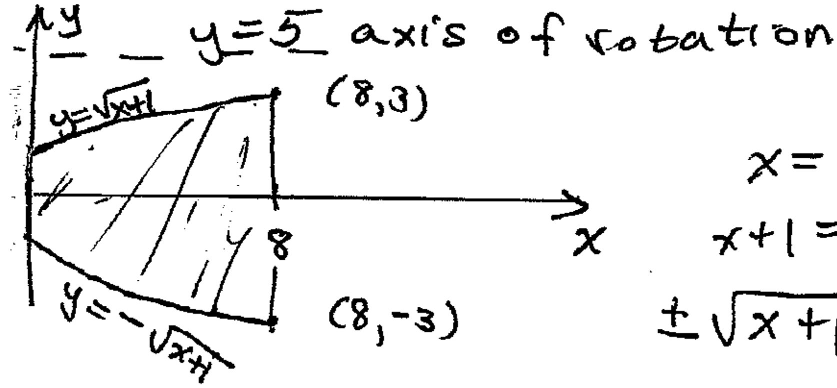
3 d)



$r = x - (-3) = x + 3$

$$2\pi \int_1^2 (x+3)(x - \ln x)\,dx \approx 31.65567$$

4. 

$x = y^2 - 1$

$x+1 = y^2$

$\pm\sqrt{x+1} = y$

Using washers:

$$\pi \int_0^8 \left(5+\sqrt{x+1}\right)^2 - \left(5-\sqrt{x+1}\right)^2 \, dx$$

$$= \pi \int_0^8 20\sqrt{x+1}\, dx = 2\pi\left(\tfrac{2}{3}\right)(x+1)^{3/2}\Big|_0^8$$

$$= \frac{4\pi}{3}\left[27-1\right] = \frac{104\pi}{3}$$

5. $\displaystyle\int_{.1}^{.2} kx\,dx = \frac{1}{2}kx^2\Big|_{.1}^{.2} = \frac{1}{2}k(.03) = .015k = 3\text{ J}$ work

$\boxed{k = 200}$

$\displaystyle\int_{.15}^{.25} 200x\,dx = 100x^2\Big|_{.15}^{.25} = \boxed{4\text{ J}}$

6. 60 lbs/30 ft =

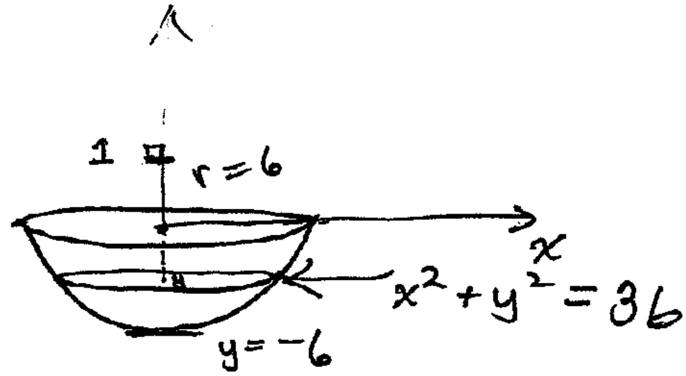
The lower 22 ft weighs 44 lbs and is raised 8 ft.

$W_1 = 44 \times 8 = 352$ ft-lbs.

$W_2 = \displaystyle\int_0^8 2y\,dy = y^2\Big|_0^8 = 64$ ft lbs.

$W = 352 + 64 = 416$ ft lbs

7.
a)



ρg is the weight density

The cross section at y is a circle of radius $x = \sqrt{36-y^2}$ with area $= \pi(36-y^2)$.

$$\pi(36-y^2)\,dy \times \rho g \times (1-y) = dW$$

↑ distance from y to the sp

$$W = \rho g \int_{-6}^{0} \pi (1-y)(36-y^2)\,dy$$

$$= \rho g \pi \int_{-6}^{0} 36 - y^2 - 36y + y^3\,dy = 468\pi\rho g$$

b) $$W = \pi\rho g \int_{-6}^{0} (36-y^2)(\overset{y-(-6)}{y+6})\,dy = 540\pi\rho g$$

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8 a) $\int x^2 \ln x\,dx$ $\quad u = \ln x \quad dv = x^2\,dx$
$\quad du = \frac{1}{x}dx \quad v = \frac{1}{3}x^3$

$$= \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2\,dx$$

$$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$$

8 b) $\int \frac{\ln x}{x}\,dx$ Substitute $u = \ln x$
$du = \frac{1}{x}\,dx$

$= \int u\,du = \frac{1}{2}u^2 + C$

$= \frac{1}{2}(\ln x)^2 + C$

8c) For $a \neq -1$,

$\int x^a \ln x\,dx$

$u = \ln x \quad dv = x^a\,dx$
$du = \frac{1}{x}\,dx \quad v = \frac{1}{a+1}x^{a+1}$

$= \frac{1}{a+1}x^{a+1}\ln x - \frac{1}{a+1}\int x^a\,dx$

$= \frac{1}{a+1}x^{a+1}\ln x - \frac{1}{(a+1)^2}x^{a+1} + C$

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8 d) $\int \arcsin x\,dx$

$u = \arcsin x \quad dv = dx$
$du = \frac{1}{\sqrt{1-x^2}}\,dx \quad v = x$

$= x\arcsin x - \int \frac{x}{\sqrt{1-x^2}}\,dx$

substitute $u = 1-x^2$
$du = -2x\,d$

$\int \frac{x}{\sqrt{1-x^2}}\,dx = \frac{-1}{2}\int \frac{du}{\sqrt{u}}$

$= -\sqrt{u} + C$

$= -\sqrt{1-x^2} +$

$= x\arcsin x + \sqrt{1-x^2} + C$

8 e) $I = \int_{-1}^{1} x \arcsin x \, dx$ $\quad u = \arcsin x \quad dv = x\,dx$

$du = \frac{1}{\sqrt{1-x^2}}\,dx \quad v = \frac{1}{2}x^2$

$$I = \frac{1}{2}x^2 \arcsin x \Big|_{-1}^{1} - \frac{1}{2}\int_{-1}^{1} \frac{x^2}{\sqrt{1-x^2}}\,dx$$

$$\left\{ \begin{aligned} x^2 &= -(1-x^2) + 1 \\ \text{so} \quad \frac{x^2}{\sqrt{1-x^2}} &= \frac{-(1-x^2)}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}} \\ &= -\sqrt{1-x^2} + \frac{1}{\sqrt{1-x^2}} \end{aligned} \right\}$$

$$I = \frac{1}{2}x^2 \arcsin x \Big|_{-1}^{1} + \frac{1}{2}\int_{-1}^{1}\sqrt{1-x^2} - \frac{1}{2}\int_{-1}^{1}\frac{1}{\sqrt{1-x^2}}$$

$$= \frac{1}{2}(\pi/2) - \frac{1}{2}(-\pi/2) + \frac{1}{2}\left(\frac{1}{2}\pi\right) - \frac{1}{2}\arcsin x \Big|_{-1}^{1}$$

$$= \pi/2 \quad + \quad \pi/4 - \frac{1}{2}\left(\frac{\pi}{2} - \left(-\pi/2\right)\right)$$

$$= \pi/4$$

8f) $\int x \sin x \, dx$

$$\boxed{\begin{array}{ll} u = x & dv = \sin x \, dx \\ du = dx & v = -\cos x \end{array}}$$

$= -x\cos x + \int \cos x \, dx$

$= -x\cos x + \sin x + C$

8g) $\int x^2 e^x \, dx$

$$\boxed{\begin{array}{ll} u = x^2 & dv = e^x \, dx \\ du = 2x\,dx & v = e^x \end{array}}$$

$= x^2 e^x - \int 2x e^x \, dx$

$$\begin{array}{ll} u = 2x & dv = e^x dx \\ du = 2dx & v = e^x \end{array}$$

$= x^2 e^x - \left[2xe^x - \int 2e^x \, dx\right]$

$= x^2 e^x - 2xe^x + 2e^x + C$

8h) $\int (\ln x)^2 \, dx$

$$\begin{array}{ll} u = (\ln x)^2 & dv = dx \\ du = \dfrac{2\ln x}{x} & v = x \end{array}$$

$= x(\ln x)^2 - \int 2\ln x \, dx$

$$\begin{array}{ll} u = 2\ln x & dv = d \\ du = \dfrac{2}{x} & v = x \end{array}$$

$= x(\ln x)^2 - \left[2x\ln x - \int 2\,dx\right]$

$= x(\ln x)^2 - 2x\ln x + 2x + C$

9, a) $I = \int_0^{\pi/2} \sin^3 x \cos^2 x \, dx = \int_0^{\pi/2} \sin x (1-\cos^2 x) \cos^2 x$

Substitute $u = \cos x$ $\quad u(0) = 1$

$du = -\sin x \, dx \quad u(\frac{\pi}{2}) = 0$

$$I = \int_1^0 (1-u^2) u^2 (-du) = \int_0^1 (1-u^2) u^2 \, du$$

$$= \int_0^1 u^2 - u^4 \, du = \frac{1}{3}u^3 - \frac{1}{5}u^5 \Big|_0^1$$

$$= \frac{1}{3} - \frac{1}{5} = \boxed{\frac{2}{15}}$$

9b)

$$\int \sin^2 x \, dx = \int \frac{1-\cos 2x}{2} \, dx$$

$$= \frac{1}{2}x - \frac{1}{4}\sin 2x + C$$

9c) $\int \sec^4 x \, dx = \int \sec^2 x \sec^2 x \, dx$

$$= \int (\tan^2 x + 1) \sec^2 x \, dx = \int u^2 + 1 \, du$$

Substitute $u = \tan x \quad du = \sec^2 x \, dx$

$$= \frac{1}{3}u^3 + u + C = \frac{1}{3}\tan^3 x + \tan x + C$$

9d) $\int \tan^4 x\, dx = \int \tan^2 x(\sec^2 x - 1)\, dx$

$= \int \tan^2 x \sec^2 x\, dx - \int \tan^2 x\, dx$

$= \int \tan^2 x \sec^2 x\, dx - \int (\sec^2 x - 1)\, dx$

$= \frac{1}{3}\tan^3 x - \tan x + x + C$

9e) $\int \sec^3 x \tan^3 x\, dx$

Use: $\boxed{\sec x \tan x = \frac{d}{dx}(\sec x)}$

$= \int \sec^2 x \tan^2 x (\sec x \tan x)\, dx$

$= \int \sec^2 x (\sec^2 x - 1) \sec x \tan x\, dx$

Substitute $u = \sec x$

$du = \sec x \tan x\, dx$

9f $\int \sin 3x \cos x \, dx$

Use the formula

$\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$

$\sin 3x \cos x = \frac{1}{2}[\sin 2x + \sin 4x]$

$\int \sin 3x \cos x \, dx = \frac{1}{2}\int \sin 2x \, dx + \frac{1}{2}\int \sin 4x \, dx$

$= -\frac{1}{4}\cos 2x - \frac{1}{8}\cos 4x + C$

9g) $\int \sin 5x \sin 3x \, dx$

Use the formula

$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

$\int \sin 5x \sin 3x \, dx$

$= \frac{1}{2}\int \cos 2x \, dx - \frac{1}{2}\int \cos 8x \, dx$

$= \frac{1}{4}\sin 2x - \frac{1}{16}\sin 8x + C$