Math 152 WIR
Solutions 8.2

1. $\int \sec^2 x \tan^3 x \, dx = \int u^3 \, du = \frac{1}{4}\tan^4 x + C$

$u = \tan x$
$du = \sec^2 x \, dx$

2. $\int \sec^3 x \tan x \, dx = \int \sec^2 x (\sec x \tan x) \, dx$

$u = \sec x$
$du = \sec x \tan x \, dx$

$= \int u^2 \, du = \frac{1}{3}u^3 + C$

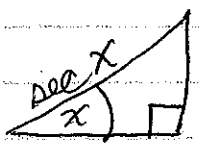
$= \frac{1}{3}\sec^3 x + C$

3. $\int \sec^4 x \, dx = \int (\tan^2 x + 1)\sec^2 x \, dx$

$u = \tan x$
$du = \sec^2 x \, dx$

$= \int (u^2 + 1) \, du = \frac{1}{3}u^3 + u + C$

$= \frac{1}{3}\tan^3 x + \tan x + C$



$\sec^2 x = 1 + \tan^2 x$

4. $\int \sec^3 x \, dx = \int \sec x \sec^2 x \, dx$ Use parts:

$u = \sec x \quad dv = \sec^2 x \, dx$
$du = \sec x \tan x \, dx \quad v = \tan x$

$\int \sec^3 x \, dx = \sec x \tan x - \int \sec x \tan^2 x \, dx$

$\tan^2 x = \sec^2 x - 1$

$= \sec x \tan x - \int \sec x (\sec^2 x - 1) \, dx$

$= \sec x \tan x - \int \sec^3 x \, dx - \int \sec x \, dx$

Add $\int \sec^3 x\,dx$ to both sides:

$$2\int \sec^3 x\,dx = \sec x\tan x - \int \sec x\,dx$$

$$= \sec x\tan x - \ln|\sec x + \tan x| + C_0$$

$$\int \sec^3 x\,dx = \frac{1}{2}\sec x\tan x - \frac{1}{2}\ln|\sec x + \tan x| + C$$

5. $$\int \tan^2 x\,dx = \int \sec^2 x - 1\,dx = \tan x - x + C$$

6. $$\int \tan^3 x\,dx = \int \tan x(\sec^2 x - 1)\,dx$$

$$= \int \tan x\sec^2 x\,dx - \int \tan x\,dx$$

$$= \frac{1}{2}\tan^2 x - \ln|\sec x| + C$$

7. $$\int \sin x\tan x\,dx = \int \frac{\sin^2 x}{\cos x}\,dx = \int \frac{1-\cos^2 x}{\cos x}\,dx$$

$$= \int \frac{1}{\cos x} - \cos x\,dx = \int \sec x - \cos x\,dx$$

$$= \ln|\sec x + \tan x| - \sin x + C$$

8. $$\int \sin^3 x\cos^2 x\,dx = \int \sin x(1-\cos^2 x)\cos^2 x\,dx$$

$$= \int (\cos^2 x - \cos^4 x)\sin x\,dx \quad \text{Substitute } u = \cos x$$

$$= \int (u^2 - u^4)(-du) = -\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C$$

$\sin x \cos x = \frac{1}{2}\sin 2x$

9. $\int \sin^2 x \cos^2 x \, dx = \frac{1}{4}\int \sin^2 2x \, dx$

$= \frac{1}{8}\int 1 - \cos 4x \, dx$ since $\sin^2 2x = \frac{1-\cos 4x}{2}$

$= \frac{1}{8}x - \frac{1}{32}\sin 4x + C$

10. $\int \sin 2x \cos 3x \, dx = \int \frac{1}{2}[\sin(2x-3x) + \sin(2x+3x)\, dx$

$= \frac{1}{2}\int (\sin(-x) + \sin 5x)\, dx$

$= \frac{1}{2}\int (-\sin x + \sin 5x)\, dx$

$= \frac{1}{2}\cos x - \frac{1}{10}\cos 5x + C$