Another Approach to Rope Problems

A weight $\frac{m}{2}$ is attached to the end of a rope which hangs freely from the top of a building. The rope weighs $\rho g$ lbs-ft and is $L$ ft long. Find the work done in pulling $d$ ft of rope to the top.

Total wt initially hanging is \( W + \frac{m}{2}Lg = T \)

\( T \) = total weight

Let \( y \) = length of rope already pulled up. The remaining weight hanging is \( T - \rho g y \). This is lifted \( dy \) ft.

\[\text{Work} = \int_0^d (T - \rho g y) \, dy\]

Ex. A 100 lb weight hangs freely from a tall building top by 30 ft of rope which weighs 60 lbs. Find the work done in pulling 20 ft of rope to the top.

\( T = 100 + 60 = 160 \) \( \rho g = \frac{60 \text{ lbs}}{30 \text{ ft}} = 2 \text{ lbs/ft} \)

\[
\text{Work} = \int_0^{20} (160 - 2y) \, dy = 2800 \text{ ft-lbs}
\]
2.4 More Tanks

1. a) A tank in the shape of a right circular cone of radius 4m and height 5m is full of water. Find the work done in pumping the water out until the depth of the water is 2m. b) Compare to 2m full and pump out all water.

2. A tank in the shape of a trapezoidal cylinder is 6m wide at the top, 4m wide at the base and 3m tall. The tank is 10 m long. Find the work done in pumping all the water out a spout 1m above the top.

3. A tank in the shape of a sphere with radius 4m is full to a depth of 6m of H\textsubscript{2}O. Find the work to pump the water out the top of the tank.