Section 13.4 Polar Coordinates

Some relations in \( x \) and \( y \) are simpler to graph if we use polar coordinates.

For example: \( x^2 + y^2 - \sqrt{x^2 + y^2} = x \)

We substitute \( x = r \cos \theta \), \( y = r \sin \theta \)

\[ x^2 + y^2 = r^2 \]

The equation in \( r \) and \( \theta \) is

\[ r^2 - r = r \cos \theta \]

\[ r - 1 = \cos \theta \]

\[ r = 1 + \cos \theta \]

We can graph this in the \((0, r)\) plane and transfer to the \((x, y)\) plane.

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\begin{array}{c}
\theta = \frac{\pi}{2} \\
r = 1 \\
p(x, y)
\end{array}
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\begin{array}{c}
\theta = 0 \\
r = 2
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The arrows indicate the direction around the curve as \( \theta \) increases.

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Graph \( r = \sin(3\theta) \)

Start with a rectangular \( \theta-r \) graph.

Each time \( r \) is 0, we are back at the origin in the \( x-y \) plane.

As \( \theta \) increases between 0 and \( \frac{\pi}{6} \), the radius increases from 0 to 1. Then the radius decreases back to 0 as \( \theta \) increases to \( \frac{\pi}{3} \).

For \( 0 \leq \theta \leq \frac{\pi}{3} \), the graph is:

For \( \frac{\pi}{3} \leq \theta \leq \frac{2\pi}{3} \), \( r \) is negative so these points are reflected through the origin.

For \( \frac{\pi}{3} \leq \theta \leq \frac{2\pi}{3} \), the graph is
The graph from $0$ to $\pi$ is

3rd loop
$\frac{2\pi}{3} \leq \theta \leq \pi$

1st loop
$0 \leq \theta \leq \frac{\pi}{3}$

2nd loop
$\frac{\pi}{3} \leq \theta \leq \frac{2\pi}{3}$

Increasing beyond $\pi$ repeats these loops.