Average Value

The average value of a continuous function, \( f(x) \), on an interval \([a, b]\) is:

\[
\frac{1}{b-a} \int_{a}^{b} f(x) \, dx
\]

If \( f(x) > 0 \), this is the area under \( f \) over \([a, b]\) divided by the base length.

\[
\frac{\text{Area}}{\text{base}} = \text{avg. height} = \frac{1}{b-a} \int_{a}^{b} f(x) \, dx
\]

Otherwise, it is \( \text{Area above base} - \text{Area below base} \),

\[
= \text{avg. value of } f \text{ on } [a, b].
\]

Examples:

1. Find the average value of \( f(x) = \sqrt{9-x^2} \) on the interval \([0, 3]\).

\( f(x) = \sqrt{9-x^2} \) is the top of a circle of radius 3.

\[
\text{Area} = \frac{1}{3} \text{area of the circle} = \frac{1}{3} \left( \frac{3\pi}{4} \right) = \frac{3\pi}{4}
\]

\[
\text{base} \quad 3
\]
2. The velocity of an object moving in a straight line is given by
\[ v(t) = 4t + 3 \text{ cm/s} \]

Find the average velocity for the time interval \([1, 3]\),
\[ \frac{1}{3-1} \int_{1}^{3} v(t) \, dt = \frac{1}{2} \int_{1}^{3} (4t + 3) \, dt \]
\[ = \frac{1}{2} \left[ 2t^2 + 3t \right]_{1}^{3} = \frac{1}{2} \left[ 18 + 9 - (2 + 3) \right] \]
\[ = 11 \text{ cm/s} \]

This is the distance traveled divided by the time change.

3. Find the avg. value of \( f(t) = t^3 + t^2 \) on the interval \([-2, 1]\).

4. Use symmetry to find the avg. value of \( f(x) = x^2 \sin x \) on \([-3, 3]\).