Integration by Partial Fractions Decomposition

All functions in the following are polynomials.

1. \[ \frac{f(x)}{Q(x)} = p(x) + \frac{r(x)}{Q(x)} \]
   where \( \deg(r(x)) < \deg(Q(x)) \)

2. \( Q(x) \) can be factored into a product of linear terms and irreducible quadratics,
   such as: \((x-1)(x-2)^3(x^2+9)^3\)

3. \( r(x) \) is a sum of terms of the form \[ \frac{A}{(x-n)^n} \text{ or } \frac{Bx+C}{(x-a)^m + b^2} \]

with these denominators being factors of \( Q(x) \), \( n, m \) positive whole numbers
To integrate:

\[
\int \frac{1}{(x-r)^n} \, dx = -\frac{1}{n-1} (x-r)^{n-1} + C \quad \text{power rule}
\]

if \( n \neq 1 \)

\[
\int \frac{1}{x-r} \, dx = \ln |x-r| + C
\]

\[
\int \frac{x}{x^2+a^2} \, dx = \frac{1}{2} \ln (x^2+a^2) + C \quad \text{by substituting} \quad u = x^2+a^2
\]

\[
\int \frac{1}{x^2+a^2} \, dx = \frac{1}{a} \arctan \frac{x}{a} + C
\]

\[
\int \frac{1}{(x^2+a^2)^m} \, dx \quad \text{Substitute} \quad x = a \tan \theta
\]