The first statement of the Fundamental Theorem of Calculus or FTC:
Given a continuous function \( f(t) \) on an interval \([a, b]\), the function defined by
\[
F(x) = \int_{a}^{x} f(t) \, dt
\]
is an antiderivative of \( f(x) \).

Why? Using the limit definition of the derivative to find \( F'(x) \), we look at a difference quotient of \( F \). The numerator of the difference quotient is \( F(x + h) - F(x) \) which is the shaded region in the graph.

Since \( f \) is continuous, \( f(t) \) is \( f(x) \) plus a tiny error \( (t) \) which approaches 0 as \( h \) approaches 0. In magnitude, this tiny error integrated from \( x \) to \( x+h \) is no larger than the maximum magnitude of the tiny errors times \( h \).

The whole difference quotient of \( F \) is
\[
\frac{F(x + h) - F(x)}{h} = \frac{1}{h} \int_{x}^{x+h} f(t) \, dt \quad \text{Now note that} \quad \frac{1}{h} \int_{x}^{x+h} f(x) \, dt = f(x)
\]
This difference quotient of \( F \) is only a tiny error from \( f(x) \) and this tiny error approaches 0 as \( h \) approaches 0.

\[
F'(x) = \lim_{h \to 0} \frac{F(x + h) - F(x)}{h} = f(x)
\]
The 2nd statement of FTC:

If \( f \) is continuous on \([a, b]\) and \( F \) is any antiderivative of \( f \) then
\[
\int_{a}^{b} f(t) \, dt = F(b) - F(a)
\]
Why? This is true by definition for the integral function defined in the first statement. If \( G \) is any antiderivative of \( f \) then \( G' = F \) so \( G - F \) is a constant, \( C \), and \( G(b) - G(a) = F(b) - F(a) \).
Examples:

1. \( \int_{1}^{4} \sqrt{t} \, dt \)

2. \( \int_{0}^{2} e^t \, dt \)

3. \( \int_{1}^{e} \frac{1}{t} \, dt \)

4. \( \int_{1}^{3} \left( t^2 - \frac{2}{t^2} - 5 \right) \, dt \)

5. \( \int_{1}^{8} \left( 5e^x + 4\sqrt{x} \right) \, dx \)

6. \( \int_{0}^{2} \left( \frac{1}{x + 1} - \frac{3}{(x + 1)^2} \right) \, dx \)

7. \( \int_{-2}^{-1} x \, dx \)

8. \( \int_{-2}^{3} (x + 1)(x - 1) \, dx \)

9. \( \int_{1}^{3} \frac{(x + 2)(x - 3)}{x} \, dx \)

Use the FTC and the Chain Rule to find the derivative of each function of \( x \).
10. $\int_1^{\sqrt{x}} (t^2 + 2) dt$

11. $\int_{\pi/6}^{\pi} \sin t \, dt$

12. $\int_{\sqrt{x}}^{\frac{\sqrt{x}}{x}} e^{u^2} \, du$