

Trigonometric Substitution

To find:

Substitute:

I $\int f(\sqrt{r^2 - x^2}) dx$

$x = r \sin \theta$
 $dx = r \cos \theta d\theta$

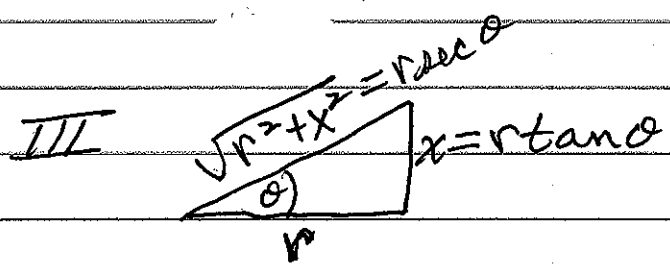
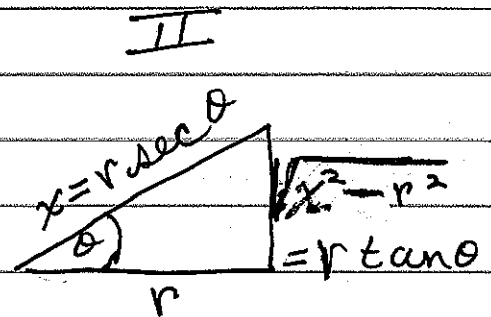
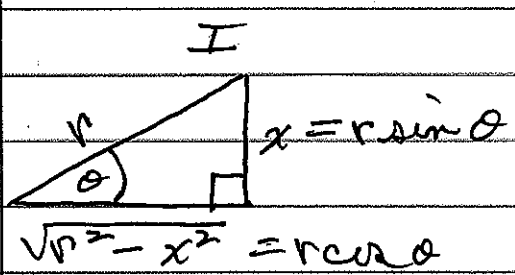
II $\int f(\sqrt{x^2 - r^2}) dx$

$x = r \sec \theta$
 $dx = r \sec \theta \tan \theta d\theta$

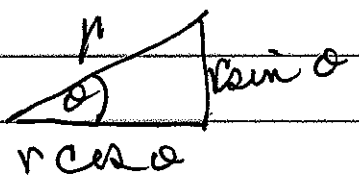
III $\int f(\sqrt{x^2 + r^2}) dx$

$x = r \tan \theta$
 $dx = r \sec^2 \theta d\theta$

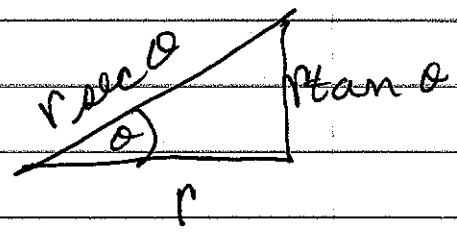
or $\int f(x^2 + r^2) dx$



The basic Δ 's



Divide by $\cos \theta \rightarrow$



Examples

1.
$$\int \frac{1}{\sqrt{4-x^2}} dx$$

2.
$$\int \frac{1}{x\sqrt{4-x^2}} dx$$

3.
$$\int \frac{x^2}{(4-x^2)^{3/2}} dx$$

4.
$$\int \frac{x^3}{\sqrt{4-x^2}} dx$$
 can be done with the substitution $u=4-x^2$ or with trig. sub.

$$5. \int \sqrt{x^2 - 9} \, dx$$

$$6. \int \frac{1}{(x^2 + 4)^2} \, dx$$

$$7. \int \frac{1}{x\sqrt{x^2 + 16}} \, dx$$

$$8. \int \frac{1}{(x^2 - 16)^{3/2}} \, dx$$

$$9. \int \frac{1}{x^2 - 16} \, dx$$