Setting up Tank Problems

1. Orient the $x$-$y$ plane in the usual way,

2. Decide how to position the vertical cross section of the tank you will use to find the area of a horizontal cross section.

Example: The tank is a trough 6 m wide at the top, 4 m high and 10 m long. The face is an isosceles triangle.

The width $x/10$ is the area of a horizontal cross-section \( A(y) = \frac{3}{2} y x/10 = 15 y \)

3. Position the spout

Example: If the spout is 1 m above the top of the trough above the tank, then the spout is at \( y = 4 + 1 = 5 \)

\[ D(y) = |\text{spout level} - y| = 5 - y \]
4. Integrate over the y-levels of the water you are pumping out.

Example: For the trough above
a) if we pump out all the water and the tank was full then
\[ \text{Work} = \rho g \int_{0}^{4} 15y(5-y) \, dy \text{ Joules} \]

b) if we only pump out the top 2m of water,
\[ \text{Work} = \rho g \int_{2}^{4} 15y(5-y) \, dy \text{ Joules} \]

c) if we pump all the water but the tank was
\[ \text{Work} = \rho g \int_{0}^{2} 15y(5-y) \, dy \text{ Joules} \]

Ex. The tank is a hemisphere of radius 3m, the spout is 1 m above the top of the tank and the tank is full to a depth of 2m. Pump out all the water.

\[ \text{Work} = \rho g \int_{-3}^{-1} (9-y^2) \, dy \]