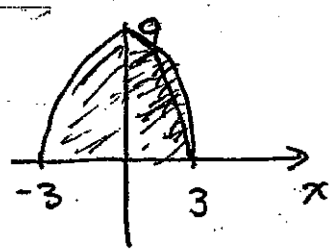
152 WIR 7.1–7.2 Solutions

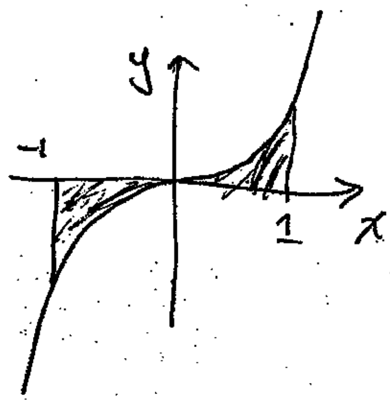
1. 

$$A = \int_{-3}^{3} 9 - x^2 \, dx$$

$$= 2\int_0^3 9 - x^2 \, dx \quad \text{by symmetry}$$

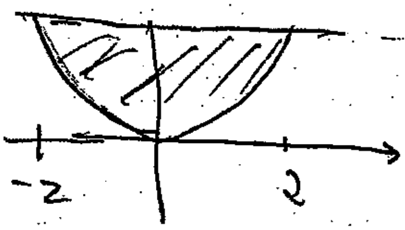
$$= 2\left[9x - \tfrac{1}{3}x^3 \Big|_0^3\right]$$

$$= 36$$

2. 

$$A = 2\int_0^1 x^3 \, dx \quad \text{by symmetry}$$

$$= \tfrac{1}{2}x^4 \Big|_0^1 = \tfrac{1}{2}$$

3. 

$$2\int_0^2 (4 - x^2)\, dx$$

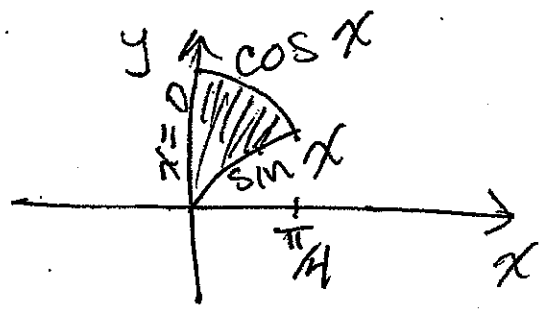
$$= 2\left[4x - \tfrac{1}{3}x^3\right]_0^2$$

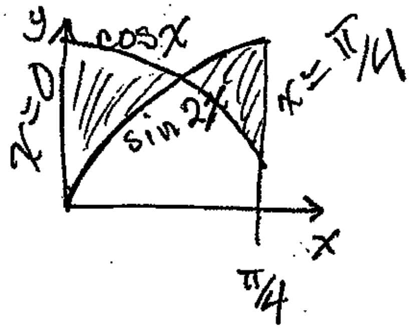
$$= 2\left[8 - 8/3\right] = \frac{32}{3}$$

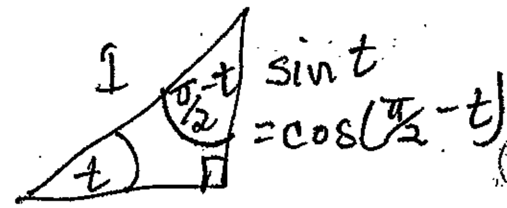
4. $$\int_0^{\pi/4} \cos x - \sin x \, dx$$

$$= \sin x + \cos x \Big|_0^{\pi/4}$$

$$= \sqrt{2} - 1$$



5. 



Solve $\cos x = \sin 2x$ $\qquad$ $\sin t = \cos(\pi/2 - t)$

$$\cos x = \cos\left(\frac{\pi}{2} - 2x\right)$$

$$x = \frac{\pi}{2} - 2x \qquad 3x = \frac{\pi}{2} \qquad x = \frac{\pi}{6}$$

$$\int_0^{\pi/6} \cos x - \sin 2x \, dx + \int_{\pi/6}^{\pi/4} \sin 2x - \cos x \, dx$$

$$= \left[\sin x + \frac{1}{2}\cos 2x \Big|_0^{\pi/6}\right] + \left[-\frac{1}{2}\cos 2x - \sin x \Big|_{\pi/6}^{\pi/4}\right]$$

$$= \frac{1}{2} + \frac{1}{4} - \left(0 + \frac{1}{2}\right) + \left[0 - \frac{\sqrt{2}}{2} - \left(-\frac{1}{4} - \frac{1}{2}\right)\right]$$

$$= \frac{1}{4} - \frac{\sqrt{2}}{2} + \frac{3}{4} = 1 - \frac{\sqrt{2}}{2}$$

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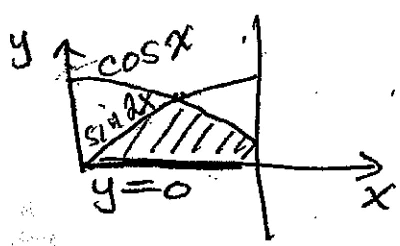
6. 

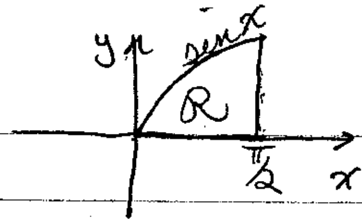
$$\int_0^{\pi/6} \sin 2x \, dx + \int_{\pi/6}^{\pi/4} \cos x \, dx$$

$$= -\frac{1}{2}\cos 2x \Big|_0^{\pi/6} + \sin x \Big|_{\pi/6}^{\pi/4}$$

$$= -\frac{1}{2}\left(\frac{1}{2}\right) - \left(-\frac{1}{2}\cdot 1\right) + \left[\frac{\sqrt{2}}{2} - \frac{1}{2}\right]$$

$$= \frac{1}{4} + \frac{\sqrt{2}}{2} - \frac{1}{2}$$

$$= \frac{\sqrt{2}}{2} - \frac{1}{4}$$

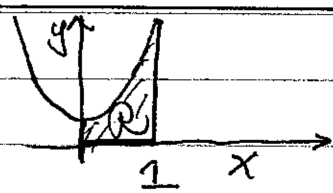
7. 

$$\pi \int_0^{\pi/2} \sin^2 x \, dx = V$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

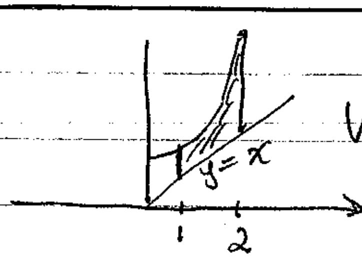
$$V = \frac{\pi}{2}\int_0^{\pi/2} (1-\cos 2x)\,dx = \frac{\pi}{2}\left[x - \frac{1}{2}\sin 2x \Big|_0^{\pi/2}\right]$$

$$= \frac{\pi}{2}\cdot\left(\frac{\pi}{2} - 0\right) = \boxed{\frac{\pi^2}{4}}$$

8. $y = e^{x^2}$ 

$$V = \pi \int_0^1 \left(e^{x^2}\right)^2 dx$$

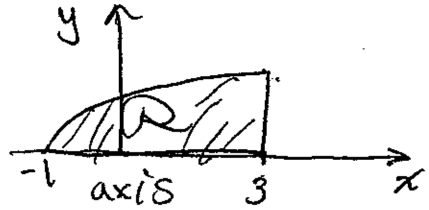
$$= \pi \int_0^1 e^{2x^2} dx$$ which must be done on a computer.

9. 

$$V = \pi \int_1^2 \left(e^{x^2}\right)^2 - x^2 \; dx$$

$$= \pi \int_1^2 e^{2x^2} dx - \pi \int_1^2 x^2 \, dx$$

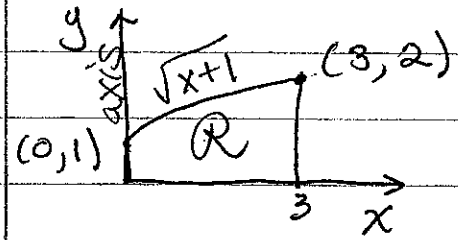
10.



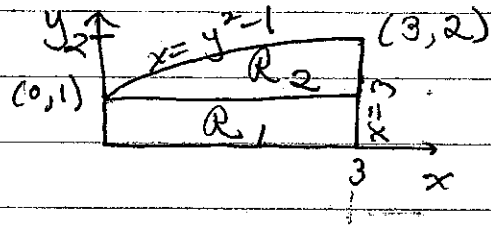
$$V = \pi \int_{-1}^{3} (x+1)\,dx = \pi\left[\tfrac{1}{2}x^2 + x \Big|_{-1}^{3}\right]$$

$$= \pi\left[\frac{9}{2} + 3 - \left(\tfrac{1}{2} - 1\right)\right] = \boxed{8\pi}$$

11.



$y = \sqrt{x+1} \quad x = y^2 - 1$



V_1 = volume from R_1 and is a right circular cylinder of radius 3, height 1.

$$V_1 = \pi(3^2)\cdot 1 = 9\pi$$

V_2 is generated by R_2

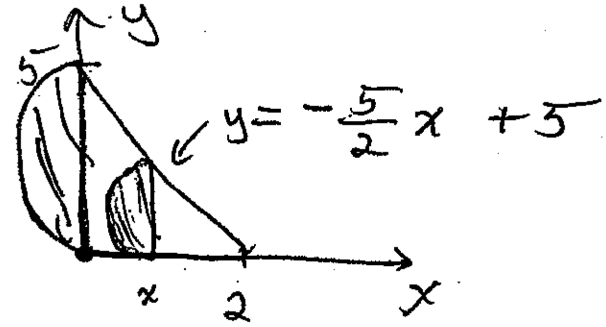
$$V_2 = \pi \int_{y=1}^{y=2} \left(3^2 - (y^2-1)^2\right) dy$$

$$= \pi \int_1^2 9 - (y^4 - 2y^2 + 1)\, dy$$

$$= \pi \int_1^2 8 - y^4 + 2y^2\, dy$$

$$= \pi\left(8y - \tfrac{1}{5}y^5 + \tfrac{2}{3}y^3 \Big|_1^2\right) \cong 20.31563$$

so $\boxed{V \approx 48.59}$

12. 

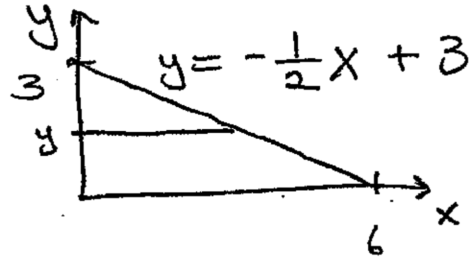
The Semicircle at x has radius $\frac{y}{2}$ and area $\pi\left(\frac{y}{2}\right)^2 = \frac{\pi}{4}y^2$

$$V = \frac{\pi}{4}\int_0^2 \left(-\tfrac{5}{2}x + 5\right)^2 dx$$

$$= \frac{\pi}{4}\left(-\frac{2}{5}\right)\left(\frac{1}{3}\right)\left(-\tfrac{5}{2}x + 5\right)^3 \Big|_0^2$$

Substitute $u = -\frac{5}{2}x + 5$ or square it all out

$$= -\frac{\pi}{30}\left[0^3 - 125\right] = \frac{125\pi}{30} = \boxed{\frac{25\pi}{6}}$$

13. 

The square at y has side $x = 6 - 2y$ and area $(6-2y)^2$.

$$\int_0^3 (6-2y)^2\, dy = \frac{1}{-2}\left(\frac{1}{3}\right)(6-2y)^3 \Big|_0^3$$

$$= \frac{-1}{6}\left(0 - 6^3\right) = \boxed{36}$$

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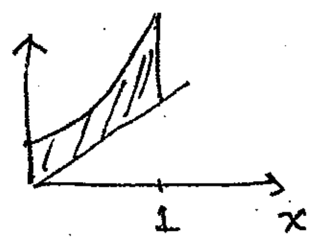
14. $\pi \int_0^1 (e^x)^2 - x^2 \, dx$ washers 

$$= \pi \int_0^1 e^{2x} - x^2 \, dx$$

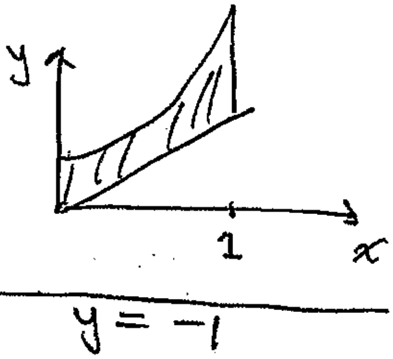
$$= \pi\left[\frac{1}{2}e^{2x} - \frac{1}{3}x^3 \Big|_0^1\right] = \pi\left[\frac{1}{2}e^2 - \frac{1}{3} - \frac{1}{2}\right]$$

$$= \pi\left[\frac{1}{2}e^2 - 5/6\right]$$

washers

15. a) $R = e^x + 1 \quad r = x + 1$

$\left(f(x) - (-1) \text{ and } g(x) - (-1) \right)$



$$V = \pi \int_0^1 (e^x+1)^2 - (x+1)^2 \, dx$$

$$= \pi \int_0^1 e^{2x} + 2e^x + 1 - (x^2 + 2x + 1) \, dx$$

$$= \pi \int_0^1 e^{2x} + 2e^x - x^2 - 2x \, dx$$

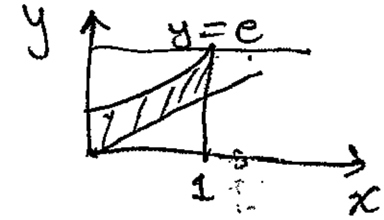
$$= \pi \left[\frac{1}{2} e^{2x} + 2e^x - \frac{1}{3} x^3 - x^2 \Big|_0^1 \right]$$

$$= \pi \left[\frac{1}{2} e^2 + 2e - \frac{1}{3} - 1 - \left(\frac{1}{2} + 2 \right) \right]$$

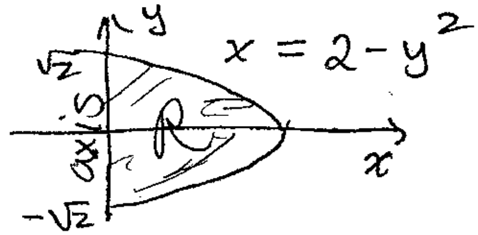
$$= \pi \left[\frac{1}{2} e^2 + 2e - \frac{23}{6} \right]$$

15 b) Setup only: $R = e - x \quad r = (e - e^x)$

$$\pi \int_0^1 (e-x)^2 - (e - e^x)^2 \, dx$$



16. Using disks



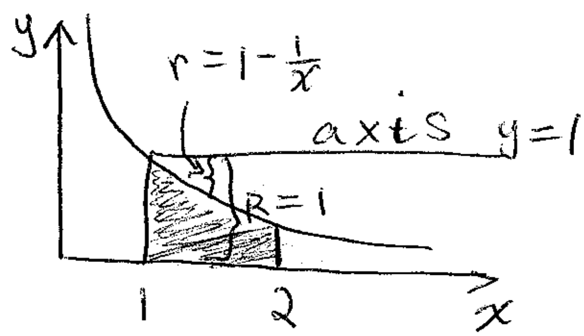
$$V = \pi \int_{-\sqrt{2}}^{\sqrt{2}} (2-y^2)^2 \, dy$$

$$= 2\pi \int_0^{\sqrt{2}} (2-y^2)^2 \, dy \quad \text{by symmetry}$$

$$= 2\pi \int_0^{\sqrt{2}} 4 - 4y^2 + y^4 \quad dy$$

$$= 2\pi \left[4y - \frac{4}{3}y^3 + \frac{1}{5}y^5 \Big|_0^{\sqrt{2}} \right] = \frac{64\sqrt{2}}{15}\pi$$

17a)

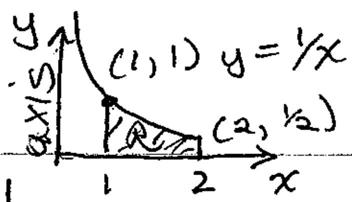


$R = 1$

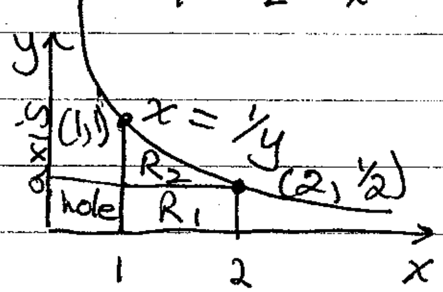
$$\pi \int_1^2 R^2 - r^2 \, dx = \pi \int_1^2 1 - \left(1 - \frac{1}{x}\right)^2 \, dx = \pi \int_1^2 \frac{2}{x} - \frac{1}{x^2} \, dx$$

$$= \pi \left[2\ln x + \frac{1}{x} \Big|_1^2 \right] = \pi \left(2\ln 2 - \frac{1}{2} \right)$$

17.



b)

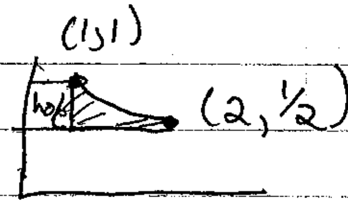


V_1

V_1 is a right circular washer with outer radius 2, inner radius 1 and height 1/2.

$$V_1 = \pi \cdot \frac{1}{2}\left(2^2 - 1^2\right) = \frac{3}{2}\pi$$

V_2 is generated by R_2:



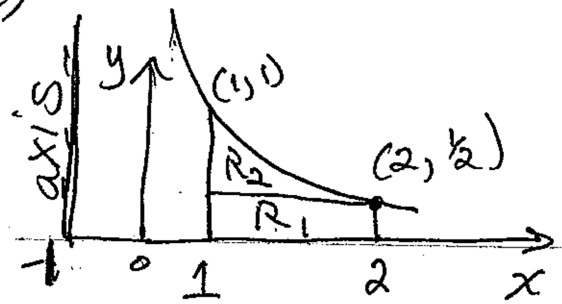
$$V_2 = \pi \int_{y=1/2}^{y=1} \left(\frac{1}{y}\right)^2 - 1^2 \, dy$$

$$= \pi \int_{1/2}^{1} \frac{1}{y^2} - 1 \, dy = \pi\left[-\frac{1}{y} - y \Big|_{1/2}^{1}\right]$$

$$= \pi\left(-2 - \left(-2 - \frac{1}{2}\right)\right) = \frac{\pi}{2}$$

$$V_1 + V_2 = \boxed{2\pi}$$

17c)



Similar to 17b) but increase the radii by 1

$$V_1 = \pi\left(\frac{1}{2}\right)(3^2 - 2^2) = \frac{5}{2}\pi \approx 7.854$$

$$V_2 = \pi \int_{1/2}^{1} \left(\frac{1}{y}+1\right)^2 - (2^2) \; dy$$

$$= \pi \int_{1/2}^{1} \left(\frac{1}{y}2 + \frac{2}{y} + 1 - 4 \; dy\right.$$

$$= \pi\left[-\frac{1}{y} + 2\ln y - 3y \Big|_{1/2}^{1} \right]$$

$$\cong .77259\pi \approx 2.42716$$

$$V \approx 10.28114$$