Solutions Math 152 WIR

10.9

(a) $c_k = \frac{h^k(1)}{k!}$ $1! = 1$ $2! = 2$ $3! = 6$ $4! = 24$

$h(x) = \ln(1+x)$ $h(1) = \ln 2$ $c_0 = \ln 2$

$h'(x) = \frac{1}{1+x}$ $h'(1) = \frac{1}{2}$ $c_1 = \frac{1}{2}$

$h''(x) = \frac{-1}{(1+x)^2}$ $h''(1) = -\frac{1}{4}$ $c_2 = -\frac{1}{8}$

$h'''(x) = \frac{2}{(1+x)^3}$ $h'''(1) = \frac{1}{4}$ $c_3 = \frac{1}{24}$

$h^4(x) = \frac{-6}{(1+x)^4}$ $h^4(1) = \frac{-6}{16} = \frac{-3}{8}$ $c_4 = -\frac{1}{64}$

$$T_4(x) = \ln 2 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{24}(x-1)^3 - \frac{1}{64}(x-1)^4$$

$$|R_4(x)| = |T_4(x) - h(x)| \le \max_{[.5,\,1.5]} |h^5(x)| \frac{(.5)^5}{5!}$$

$h^5(x) = \frac{24}{(1+x)^5} \le \frac{24}{(1.5)^5}$ (Make $|1+x|$ as small as it can be on $[.5, 1.5]$

$$|R_4(x)| \le \frac{\cancel{24}}{\left(\frac{3}{2}\right)^5} \cdot \frac{\left(\frac{1}{2}\right)^5}{5 \times \cancel{24}}$$

$$= \frac{1}{3^5 \times 5} = \frac{1}{1215}$$

2 a) The Maclaurin series for e^x is

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Going out to x^3, $T_{3,f}(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$

$|f^{4}(x)| = e^x \le e^{.2}$ on $[-.2, .2]$

$$|R_{3,f}(x)| \le \frac{e^{.2}(.2)^4}{24} \qquad 24 = 4!$$

$$= \frac{2e^{.2}}{3} \times 10^{-4}$$

b) $g(x) = x^2 e^x = \sum_{n=0}^{\infty} \frac{x^{n+2}}{n!}$

$$T_{5,g}(x) = x^2 + x^3 + \frac{x^4}{2} + \frac{x^5}{6}$$

$$|R_{5,g}(x)| \le x^2 \, \frac{2e^{.2}}{3} \times 10^{-4} \le \frac{8}{3} e^{.2} \times 10^{-6}$$

2c) $h(x) = e^{-x^2} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{n!}$

$$T_{6,h}(x) = 1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6}$$

Using the alternating series remainder, $b_n \downarrow 0$,

$$|R_{6,h}(x)| \le \frac{x^8}{4!} \le \frac{(.2)^4}{24} = \frac{2}{3} \times 10^{-4}$$

3. $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$

$T_5(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5$

Since $T_5(x) = T_6(x)$ in this case and

$\left|\frac{d^7}{dx^7}\sin x\right| = \left|\frac{d^3}{dx^3}\sin x\right| = |-\cos x| \leq 1$ on $|x| \leq \frac{\pi}{3}$

$|R_5(x)| \leq \frac{\left(\frac{\pi}{3}\right)^7}{7!}$ Same as the alt. series remainder. The series alternates for all x.

4. $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ so

$\cos\sqrt{x} = \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{(2n)!}$ $*$

$T_3(x) = 1 - \frac{x}{2} + \frac{x^2}{24} - \frac{x^3}{720}$ on $\left[0, \left(\frac{\pi}{6}\right)^2\right]$

The series $*$ is alternating for $x > 0$ so

$|R_3(x)| \leq b_4 = \frac{x^4}{8!} \leq \frac{\left(\frac{\pi}{6}\right)^8}{8!}$

5. $c_n = \left(\frac{d^n}{dx^n} \sin x \Big|_{x=\pi/3} \right) \cdot \frac{1}{n!}$

$f(x) = \sin x \qquad \sin \pi/3 = \frac{\sqrt{3}}{2} \qquad c_0 = \frac{\sqrt{3}}{2}$

$f'(x) = \cos x \qquad \cos \frac{\pi}{3} = \frac{1}{2} \qquad c_1 = \frac{1}{2}$

$f''(x) = -\sin x \qquad -\sin \pi/3 = -\frac{\sqrt{3}}{2} \qquad c_2 = -\frac{\sqrt{3}}{4}$

$f'''(x) = -\cos x \qquad -\cos \frac{\pi}{3} = -\frac{1}{2} \qquad c_3 = -\frac{1}{12}$

$f^{4}(x) = \sin x \qquad \sin \pi/3 = \frac{\sqrt{3}}{2} \qquad c_4 = \frac{\sqrt{3}}{48}$

$$T_4(x) = \frac{\sqrt{3}}{2} + \frac{1}{2}\left(x - \frac{\pi}{3}\right) - \frac{\sqrt{3}}{4}\left(x - \frac{\pi}{3}\right)^2 - \frac{1}{12}\left(x - \pi/3\right)^3 + \frac{\sqrt{3}}{48}\left(x - \pi/3\right)^4$$

$\frac{d^5}{dx^5}(\sin x) = \cos x$ and $|\cos x| \le \frac{\sqrt{3}}{2}$

on $[\pi/6, \pi/2]$.

$$|R_4(x)| \le \frac{\sqrt{3}}{2} \frac{1}{120} |x - \pi/3|^5 \le \frac{\sqrt{3}}{240}\left(\frac{\pi}{6}\right)^5$$

6. $\ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n}$ as done before.

$\ln(1.4) = \sum_{n=0}^{\infty} (-1)^n \frac{(.4)^n}{n}$ Using the alternating series remainder $|R_n| \le \frac{(.4)^{n+1}}{n+1} \le 10^{-3}$ if $n = 5$. (Using a calculator.)

11.1

7. $x^2+y^2+z^2 = 8x-6y+10z-2$

$x^2-8x+y^2+6y+z^2-10z = -2$

Completing the squares

$(x-4)^2+(y+3)^2+(z-5)^2 = -2+16+9+25$
$= 48$

$C(4,-3,5)$ $r=\sqrt{48}=4\sqrt{3}$

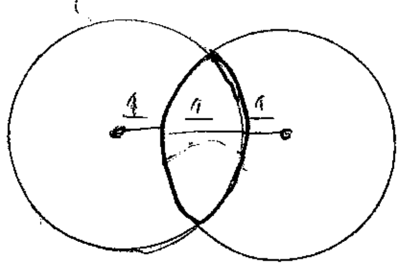
8. $x^2+y^2+z^2=4$ is a sphere centered at $(0,0,0)$ with radius 2.

$x^2+y^2+z^2+4x-2y+4z = -5$

$(x+2)^2+(y-1)^2+(z+2)^2 = -5+4+1+4$
$= 4$

is a sphere centered at $(-2,1,-2)$ with radius 2.

The distance between the centers $C_1(0,0,0)$ and $C_2(-2,1,-2)$ is $\sqrt{4+1+4} = 3$



A two sided cap of a sphere.

9. Find $|KL|$, $|KM|$, and $|LM|$. If the pts are collinear, two of these add to make the third.

$|KL| = \sqrt{1^2 + (2-3)^2 + (-2+4)^2} = \sqrt{6}$

$|KM| = \sqrt{3^2 + 3^2 + 5^2} = \sqrt{43}$

$|LM| = \sqrt{(3-1)^2 + 2^2 + (1+2)^2} = \sqrt{17}$

$\sqrt{6} + \sqrt{17} > \sqrt{43}$ since when squaring:

$6 + 2\sqrt{6}\sqrt{17} + 17 \overset{?}{\geq} 43$

$2\sqrt{6}\sqrt{17} \overset{?}{\geq} 20$

$2\sqrt{102} \overset{?}{>} 2(10)$ yes.

Not collinear. They make an obtuse triangle.

10. $P(x,y,z)$ $A(-1,5,3)$ $B(6,2,-2)$

Find all such P so $|AP| = 2|PB|$

$|AP| = \sqrt{(x+1)^2 + (y-5)^2 + (z-3)^2}$

$|PB| = \sqrt{(x-6)^2 + (y-2)^2 + (z+2)^2}$

Solve $|AP|^2 = 4|PB|^2$ All x^2, y^2 and z^2 have coefficient 3:

$x^2 + 2x + 1 + y^2 - 10y + 25 + z^2 - 6z + 9$

$= 4[x^2 - 12x + 36 + y^2 - 4y + 4 + z^2 + 4z + 4]$

$3x^2 + 3y^2 + 3z^2 - 50x - 6y + 22z + 4(36+4+4)$

$1 \neq 25 + 9 = 35$

$$3x^2 + 3y^2 + 3z^2 - 50x - 6y + 22z = 35 - 4(44)$$

$$= -141$$

$$x^2 + y^2 + z^2 - \frac{50}{3}x - 2y + \frac{22}{3}z = -\frac{141}{3}$$

$$\left(x - \frac{25}{3}\right)^2 + (y-1)^2 + \left(z + \frac{11}{3}\right)^2 =$$

$$\frac{-141}{3} + \left(\frac{25}{3}\right)^2 + 1 + \left(\frac{11}{3}\right)^2$$

$$= \frac{332}{9}$$

$C\left(\frac{25}{3}, 1, \frac{-11}{3}\right)$ $\quad r = \frac{\sqrt{332}}{3}$