Math 152 WIR Fall '09
11.2, 11.3 solutions

11.2

1 a) $\vec{a} = \langle 6, 0, 2 \rangle \quad \vec{b} = \langle 5, 3, -2 \rangle$

$|\vec{a}||\vec{b}|\cos\theta = \vec{a}\cdot\vec{b} = 6\cdot 5 + 0\cdot 3 + 2\cdot(-2)$
$= 26$

$|\vec{a}| = \sqrt{36+4} = 2\sqrt{10} \qquad |\vec{b}| = \sqrt{25+9+4}$
$= \sqrt{38}$

$$\cos\theta = \frac{26}{2\sqrt{10}\sqrt{38}} = \frac{13}{\sqrt{380}}$$

$$\theta = \cos^{-1}\left(\frac{13}{\sqrt{380}}\right)$$

b) $\vec{a}\cdot\vec{b} = 1\cdot 2 + 1\cdot 0 + 0(-3) = 2$
$|\vec{a}| = \sqrt{2} \quad |\vec{b}| = \sqrt{13}$

$$\theta = \cos^{-1}\left(\frac{2}{\sqrt{2}\sqrt{13}}\right) = \cos^{-1}\left(\frac{2}{\sqrt{26}}\right)$$

2. If parallel, $|\vec{a}\cdot\vec{b}| = |\vec{a}||\vec{b}|$ or $(\vec{a}\cdot\vec{b})^2 = |\vec{a}|^2|\vec{b}$
If perpendicular, $\vec{a}\cdot\vec{b} = 0$.

a) $\vec{a}\cdot\vec{b} = -4+10-6 = 0$ ... endicular (orthogonal)

b) $\vec{a}\cdot\vec{b} = -6-54-24$
$= -84$

$|\vec{a}| = \sqrt{4+36+16}$
$= \sqrt{56}$
$|\vec{b}| = \sqrt{9+81+36} = \sqrt{126}$

$56 \times 126 = 81^2$ so parallel

Alternatively: $\vec{a} = 2(\vec{i} + 3\vec{j} - 2\vec{k})$
$\vec{b} = -3(\vec{i} + 3\vec{j} - 2\vec{k})$ > parallel

$a_1 = |\vec{a}|\cos\alpha \quad a_2 = |\vec{a}|\cos\beta \quad a_3 = |\vec{a}|\cos\gamma$

3. $|\vec{a}|^2 = a_1^2 + a_2^2 + a_3^2 = |\vec{a}|^2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

so $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2\left(\frac{\pi}{4}\right) + \cos^2\left(\pi/3\right) = \frac{1}{2} + \frac{1}{4} = 3/4$

so if θ is the 3rd direction angle
then $\cos^2\theta = 1 - 3/4 = 1/4$

$\cos\theta = \pm\frac{1}{2} \qquad \cos\theta = \frac{1}{2} \quad \boxed{\theta = \pi/3}$

take positive

4. Find the scalar projection of $\vec{b}$ onto $\vec{a}$ or

$\text{comp}_{\vec{a}}\vec{b} = \frac{\vec{b}\cdot\vec{a}}{|\vec{a}|}$

a) $\text{comp}_{\vec{a}}\vec{b} = \frac{6-3}{\sqrt{9+1}} = \frac{3}{\sqrt{10}}$

The vector projetion of $\vec{b}$ onto $\vec{a}$ is

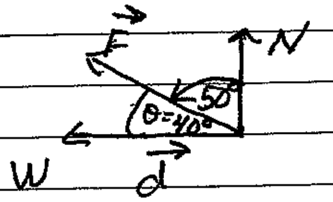
is $(\text{comp}_{\vec{a}}\vec{b})\frac{\vec{a}}{|\vec{a}|} = \frac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\vec{a} = \text{proj}_{\vec{a}}\vec{b}$

$\text{proj}_{\vec{a}}\vec{b} = \frac{3}{10}(3\vec{i} - \vec{j}) = \langle .9, -.3\rangle$

b) $\text{comp}_{\vec{a}}\vec{b} = \frac{2-18-2}{\sqrt{4+9+1}} = \frac{-18}{\sqrt{14}}$

$\text{proj}\,\vec{b} = -\frac{18}{14}(2\vec{i} - 3\vec{j} + \vec{k}) = \frac{9}{7}(-2\vec{i} + 3\vec{j} - \vec{k})$

5. $\vec{F} \cdot \vec{d} = 20 \times 4 \cos\theta = 80\cos(40^\circ)$



11.3

6. $\vec{a} \times \vec{b}$ is orthogonal to both $\vec{a}$ and $\vec{b}$.

Find $\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$ and $-\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

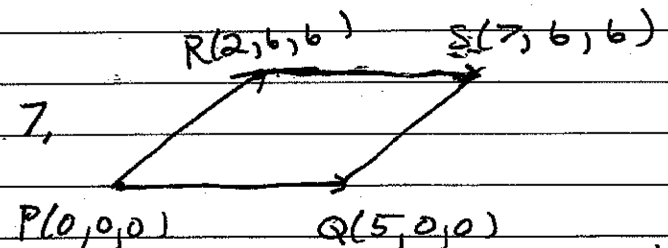
$\vec{a} = \vec{i} + \vec{j} \qquad \vec{b} = \vec{i} - \vec{j} + \vec{k}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 1 & -1 & 1 \end{vmatrix} = \vec{i}(1-0) - \vec{j}(1-0) + \vec{k}(-1-1)$$

$$= \vec{i} - \vec{j} - 2\vec{k}$$

$$|\vec{a}\times\vec{b}| = \sqrt{1+1+4} = \sqrt{6}$$

$$\frac{1}{\sqrt{6}}\vec{i} - \frac{1}{\sqrt{6}}\vec{j} - \frac{2}{\sqrt{6}}\vec{k} \quad \text{and} \quad \frac{-1}{\sqrt{6}}\vec{i} + \frac{1}{\sqrt{6}}\vec{j} + \frac{2}{\sqrt{6}}\vec{k}$$

7. 

$$\text{Area} = |\vec{PQ} \times \vec{PR}| = \left\| \begin{matrix} i & j & k \\ 5 & 0 & 0 \\ 2 & 6 & 6 \end{matrix} \right\| =$$

$$= |\vec{i}(0) - \vec{j}(30) + \vec{k}(30)| = 30\sqrt{2}$$

8. Volume $= \left| \vec{a} \cdot (\vec{b} \times \vec{c}) \right|$

$$= \left| \begin{vmatrix} 2 & -3 & -2 \\ 1 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix} \right|$$

$$= |2(-3) - 3(3) + (-2)(2)|$$

$$= |-6 - 9 - 4| = 19$$

9. $\overrightarrow{PQ} = \langle 2, 3, 3 \rangle = \vec{a}$

$\overrightarrow{PR} = \langle -1, -1, -1 \rangle = \vec{b}$

$\overrightarrow{PS} = \langle 6, -2, 2 \rangle = \vec{c}$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} 2 & 3 & 3 \\ -1 & -1 & -1 \\ 6 & -2 & 2 \end{vmatrix} = 2(-4) - 3(-2+6) + 3(2+6)$$

$$= -8 - 12 + 24$$

$$= 4$$

10. $\vec{a} = \overrightarrow{PQ} = \langle 1, 4, 5 \rangle$ $\vec{b} = \overrightarrow{PR} = \langle 2, -1, 1 \rangle$

$\vec{c} = \overrightarrow{PS} = \langle 5, 2, 7 \rangle$

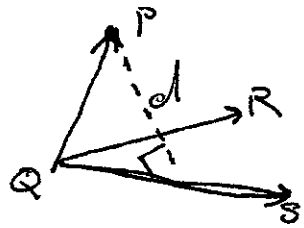
$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} 1 & 4 & 5 \\ 2 & -1 & 1 \\ 5 & 2 & 7 \end{vmatrix} = 1(-7-2) - 4(14-5) + 5(4+5)$$

$$= -9 - 36 + 45$$

$$= 0$$

Shows $\vec{a}$ is orthogonal to the normal to the plane of $\vec{b}$ and $\vec{c}$.

11.



$d = \left|\text{comp}_{\vec{n}}\, \overrightarrow{QP}\right|$ where $\vec{n}$ is orthogonal to the plane of $\underset{\vec{a}}{\overrightarrow{QR}}$ and $\underset{\vec{b}}{\overrightarrow{QS}}$

so $d = \left|\text{comp}_{\vec{a}\times\vec{b}}\, \vec{c}\right| = \left|\dfrac{\vec{c}\cdot(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|}\right|$

But $\left|\vec{c}\cdot(\vec{a}\times\vec{b})\right| = \left|\vec{a}\cdot(\vec{b}\times\vec{c})\right|$

so $d = \left|\dfrac{\vec{a}\cdot(\vec{b}\times\vec{c})}{|\vec{a}\times\vec{b}|}\right|$