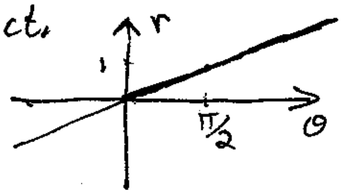
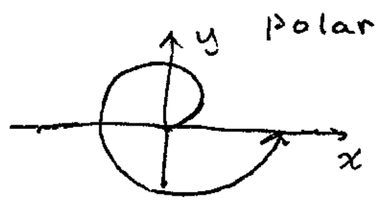
Math 152 WIR Fall '09
13.4 solutions

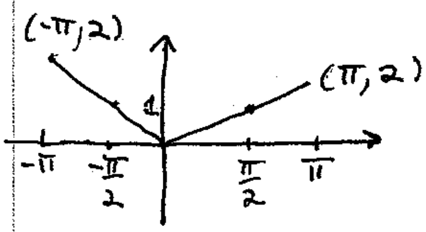
1. $r = \frac{2}{\pi}\theta$ on $[0, 2\pi]$

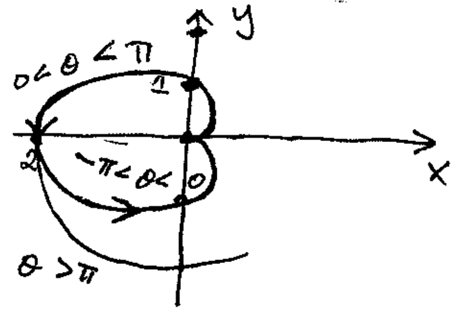
rect.



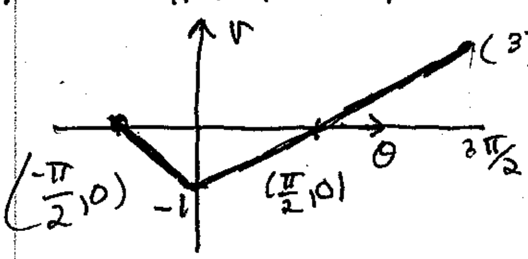


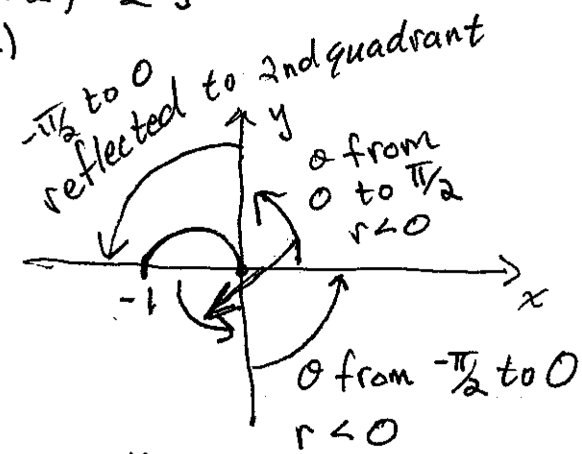
a) $r = \frac{2}{\pi}|\theta|$ on $[-\pi, \pi]$



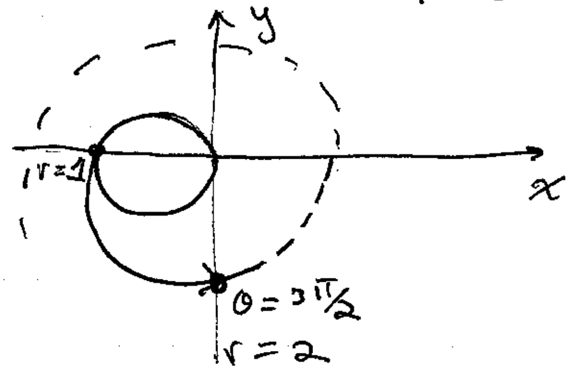


b) $r = \frac{2}{\pi}|\theta| - 1$ on $[-\frac{\pi}{2}, \frac{3\pi}{2}]$



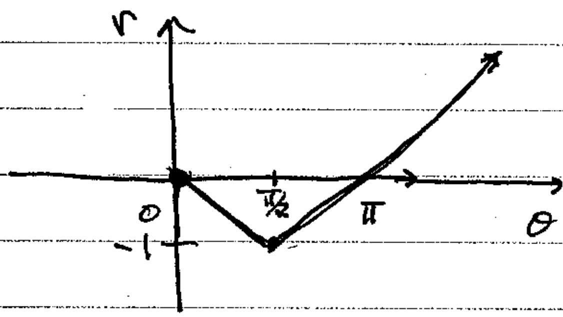


where $r<0$, reflect across the origin in the polar graph.

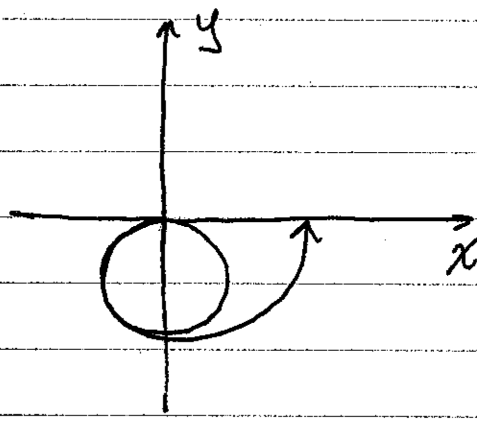


1c) $r = \frac{2}{\pi}\left|\theta - \frac{\pi}{2}\right| - 1$ on $[0, 2\pi]$ See 1b.

In rectangular (θ, r) coordinates, the graph is shifted $\frac{\pi}{2}$ units to the right. The polar graph is rotated counterclockwise by $\frac{\pi}{2}$ radians.



rectangular graph of 1b shifted right by $\frac{\pi}{2}$



polar graph of 1b rotated counterclockwise $\pi/2$ radians.

2. $r = 2\sin\theta + 2\cos\theta$

Multiplying by r:

$r^2 = 2r\sin\theta + 2r\cos\theta$

Convert to rectangular (x, y):

$x^2 + y^2 = 2y + 2x$

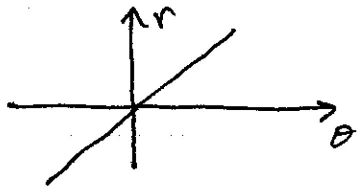
Complete the squares:

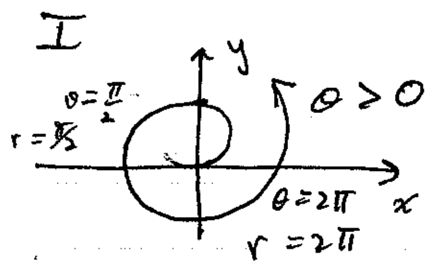
$x^2 - 2x + y^2 - 2y = 0$

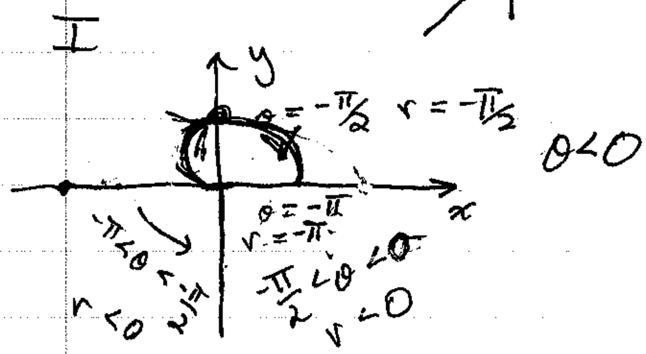
$(x-1)^2 + (y-1)^2 = 2$

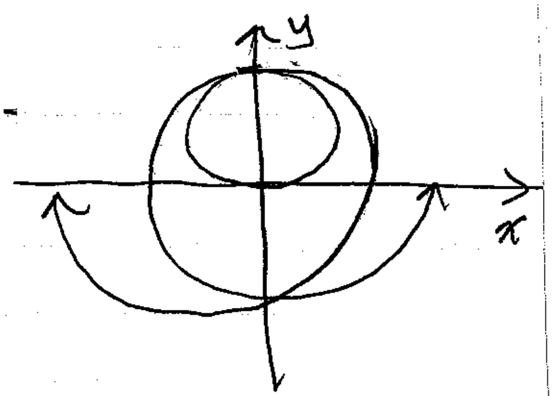
Circle centered at $(x,y) = (1,1)$ of radius $\sqrt{2}$.

3. $r = \theta$



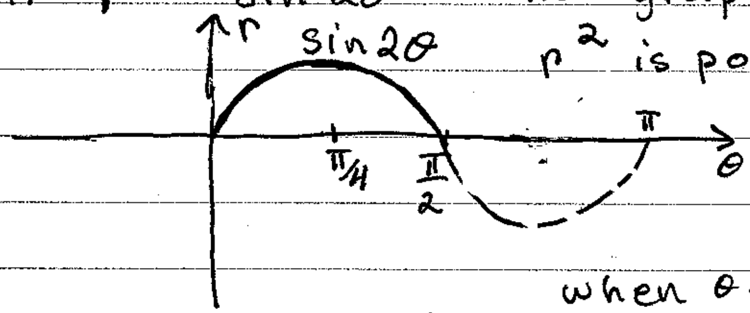






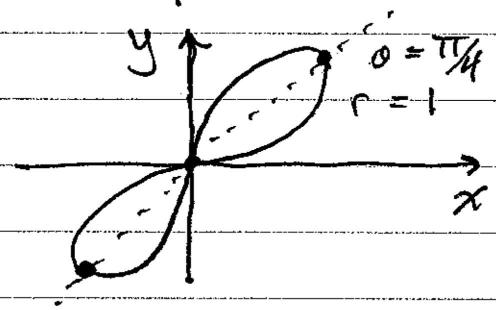
4. $r^2 = \sin 2\theta$ The (rectangular) graph of r^2:

r^2 is positive so only graph on $[0, \pi/2]$



when $\theta = \pi/4$

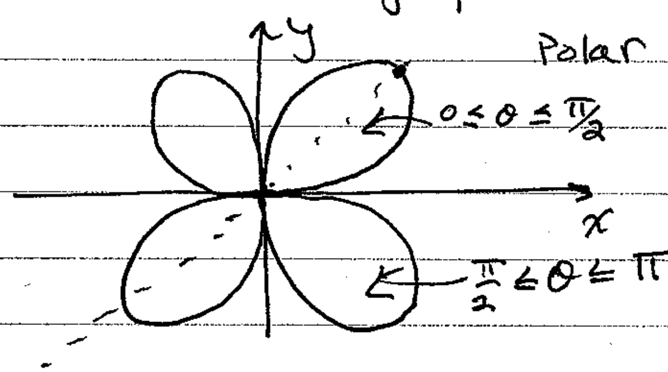
$r^2 = 1$ so $r = 1$ or $r = -1$



$\theta = \pi/4$ $r = -1$

5. $r = \sin 2\theta$ graph on $[0, \pi]$

Polar



6. $r = \tan\theta$ As $\theta \to -\frac{\pi}{2}^{+}$ $r \to -\infty$. $\theta \to \frac{\pi}{2}^{-}$, $r \to +\infty$

$\theta \neq \frac{\pi}{2}$ What is the vertical asymptote?

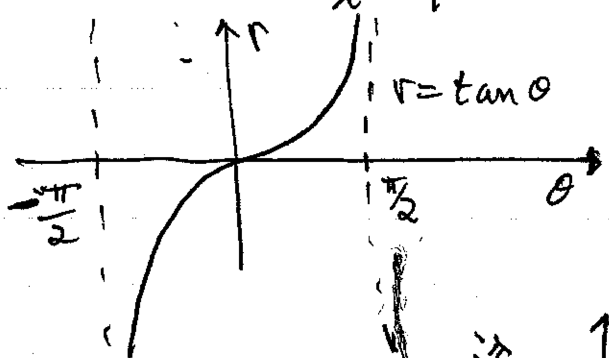
Convert to (x, y):

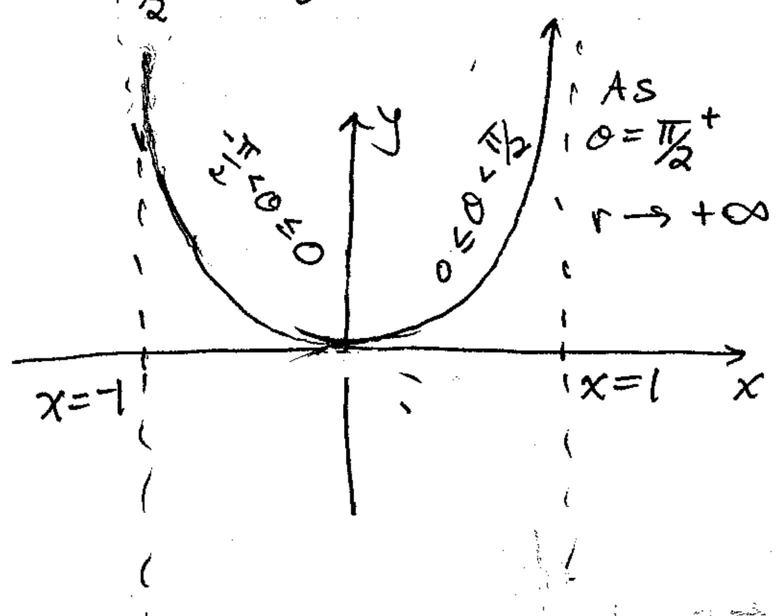
$$r = \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{r\sin\theta}{r\cos\theta} = \frac{y}{x}$$

$$\sqrt{x^2+y^2} = \frac{y}{x} \qquad x^2+y^2 = \frac{y^2}{x^2}$$

Can be simplified to $x^4 + (x^2-1)y^2 = 0$

or $y^2 = \frac{-x^4}{x^2-1}$ V. A. $x = 1$ $x = -1$

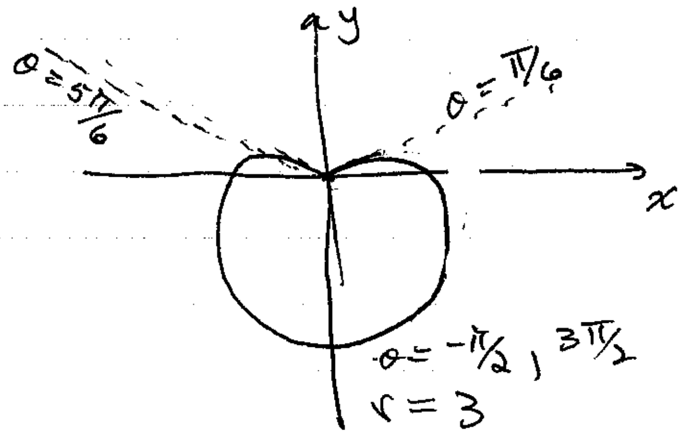




7. $r = 1 - 2\sin\theta$

The max of r occurs when $\sin\theta = -1$, at $\theta = -\pi/2$ and $3\pi/2$

$r = 0$ when $\sin\theta = \frac{1}{2}$ $\theta = \pi/6, 5\pi/6$

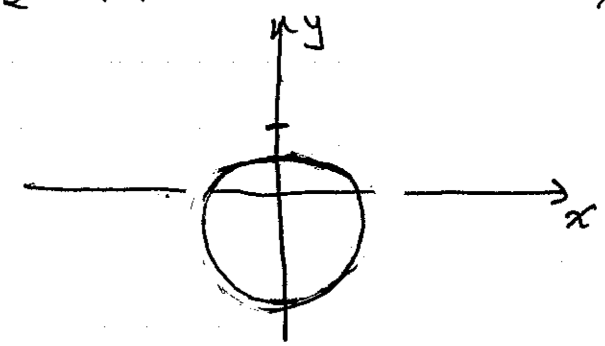


8. $r = 1 - \frac{1}{2}\sin\theta$ Now r can never be 0.

$\frac{1}{2} \leq 1 - \frac{1}{2}\sin\theta \leq 3/2$

$r = 1/2$ if $\sin\theta = 1$ $\theta = \pi/2$

$r = 3/2$ if $\sin\theta = -1$ $\theta = 3\pi/2$



9. $r = 1 + 3\cos\theta$ $\quad -2 \le r \le 4$

$r = -2$ if $\theta = \pi$ $\quad r = 4$ if $\theta = 0$

$r = 0$ if $\cos\theta = -1/3$ $\quad \theta = \arccos(-1/3) = \alpha$