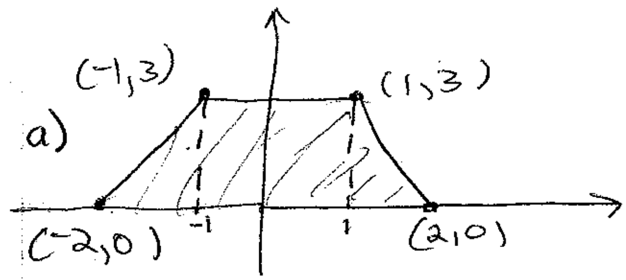
Math 152 WIR Solutions

7.5 7.5, 8.1, 8.2

1. a)

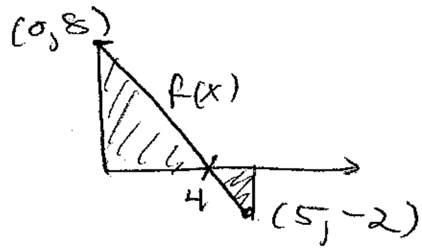


Since $f \geq 0$, $\frac{1}{2-(-2)} \int_{-2}^{2} f(x)\,dx = \frac{\text{Area}}{4}$

$\text{Area} = \frac{1}{2}(1 \times 3) + 2 \times 3 + \frac{1}{2}(1 \times 3) = 9$

avg value $= \boxed{9/4}$

b)



$f(x) = 8 - 2x$

$\frac{1}{5} \int_0^5 f(x)\,dx = \frac{1}{5}[A1 - A2]$

$= \frac{1}{5}\left[\frac{1}{2}(4 \times 8) - \frac{1}{2}(1 \times 2)\right]$

$= \frac{1}{5}(16 - 1) = \boxed{3}$

2. $\frac{1}{2r} \int_{-r}^{r} \sqrt{r^2 - x^2}\,dx = \frac{1}{2r}$ (area of a semicircle of radius r)

$= \frac{1}{2r} \cdot \frac{1}{2}\pi r^2 = \boxed{\frac{\pi r}{4}}$

3 a) $\frac{1}{\pi/2} \int_0^{\pi/2} \sin x\,dx = \frac{2}{\pi}\left[-\cos x \Big|_0^{\pi/2}\right] = \boxed{\frac{2}{\pi}}$

b) $\frac{1}{\pi/4} \int_0^{\pi/4} \sec^2 x\,dx = \frac{4}{\pi} \tan x \Big|_0^{\pi/4} = \boxed{\frac{4}{\pi}}$

3c) $2\int_0^{1/2} \frac{x}{\sqrt{1-x^2}}\,dx$

Substitute $u = 1-x^2$ $\quad u(0)=1 \quad u(\tfrac{1}{2}) = \tfrac{3}{4}$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$$\cancel{2}\int_1^{3/4} \left(-\frac{1}{\cancel{2}}\right)\left(u^{-1/2}\right)du = -\int_1^{3/4} u^{-1/2}\,du$$

$$= -2u^{1/2}\Big|_1^{3/4} = -2\frac{\sqrt{3}}{2} + 2 = \boxed{2\left(1-\frac{\sqrt{3}}{2}\right)}$$

8.1

4. a) $\int \ln x\,dx \qquad = x\ln x - \int dx$

$u = \ln x \quad dv = dx$

$du = \frac{1}{x}dx \quad v = x$

$= x\ln x - x + C$

b) $\int \sqrt{x}\ln x\,dx \qquad = \frac{2}{3}x^{3/2}\ln x - \int \frac{2}{3}x^{1/2}\,dx$

$u = \ln x \quad dv = \sqrt{x}\,dx$

$du = \frac{1}{x}dx \quad v = \frac{2}{3}x^{3/2}$

$= \frac{2}{3}x^{3/2}\ln x - \frac{4}{9}x^{3/2} + C$

c) $\int xe^{2x}\,dx \qquad = \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\,dx$

$u = x \quad dv = e^{2x}dx$

$du = dx \quad v = \frac{1}{2}e^{2x}$

$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$

4 d) $\int x\sqrt{x+1}\,dx$ can be done by substituting $u = x+1$ or by parts.

$u = x$ $\quad dv = \sqrt{x+1}\,dx$

$du = dx$ $\quad v = \frac{2}{3}(x+1)^{3/2}$

$$\int x\sqrt{x+1}\,dx = \frac{2}{3}x(x+1)^{3/2} - \int \frac{2}{3}(x+1)^{3/2}\,dx$$

$$= \frac{2}{3}x(x+1)^{3/2} - \frac{4}{15}(x+1)^{5/2} + C$$

The answer from the substitution looks different but differs only by a constant.

4e) $\int e^{2x}\sin x\,dx$

$u = e^{2x}$ $\quad dv = \sin x\,dx$ (The other choice also works)

$du = 2e^{2x}dx$ $\quad v = -\cos x$

$$\int e^{2x}\sin x\,dx = -e^{2x}\cos x - \int -2e^{2x}\cos x\,dx$$

$$= -e^{2x}\cos x + \int 2e^{2x}\cos x\,dx$$

$u = 2e^{2x}$ $\quad dv = \cos x\,dx$ Use same type choice

$du = 4e^{2x}dx$ $\quad v = \sin x\,dx$

$$= -e^{2x}\cos x + 2e^{2x}\sin x - 4\int e^{2x}\sin x\,dx$$

add to both sides

$$5\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2e^{2x}\sin x + C_0$$

$$\int e^{2x}\sin x\,dx = -\frac{1}{5}e^{2x}\cos x + \frac{2}{5}e^{2x}\sin x + C$$

$(C_0/5 = C)$

4 f) $\int \frac{\ln x}{x^2}\,dx = -\frac{1}{x}\ln x + \int \frac{1}{x^2}\,dx$

$u = \ln x \qquad dv = \frac{1}{x^2}\,dx$

$du = \frac{1}{x}\,dx \qquad v = -\frac{1}{x}$

$= -\frac{1}{x}\ln x - \frac{1}{x} + C$

4 g) $\int \arcsin x\,dx = x\arcsin x - \int \frac{x}{\sqrt{1-x^2}}\,dx$

$u = \arcsin x \qquad dv = dx$

$du = \frac{dx}{\sqrt{1-x^2}} \qquad v = x$

Substitute

$w = 1 - x^2$

$dw = -2x\,dx$

$-\frac{1}{2}\,dw = x\,dx$

$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\int w^{-1/2}\,dw = -w^{1/2} = -\sqrt{1-x^2}$

$\int \arcsin x\,dx = x\arcsin x + \sqrt{1-x^2} + C$

4 h) $\int \arctan x\,dx = x\arctan x - \int \frac{x}{1+x^2}\,dx$

$u = \qquad dv = dx$

$du = \frac{1}{1+x^2}\,dx \qquad v = x$

$= x\arctan x - \frac{1}{2}\ln(1+x^2) + C$

(the last done by substitutir $w = 1+x^2$)

8.2

5. a) $\int \sec^2 x \tan^3 x \, dx = \int u^3 \, du = \frac{1}{4}\tan^4 x + C$

$u = \tan x$

$du = \sec^2 x \, dx$

b) $\int \sec^3 x \tan x \, dx = \int \sec^2 x (\sec x \tan x) \, dx$

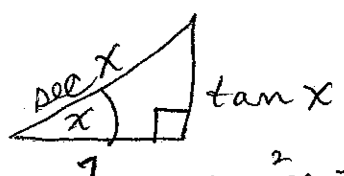
$u = \sec x$ $\qquad = \int u^2 \, du = \frac{1}{3}u^3 + C$

$du = \sec x \tan x \, dx$

$= \frac{1}{3}\sec^3 x + C$

c) $\int \sec^4 x \, dx = \int (\tan^2 x + 1)\sec^2 x \, dx$

$\left.\begin{array}{l} u = \tan x \\ du = \sec^2 x \, dx \end{array}\right] = \int (u^2 + 1) \, du = \frac{1}{3}u^3 + u + C$



$= \frac{1}{3}\tan^3 x + \tan x + C$

$\sec^2 x = 1 + \tan^2 x$

d) $\int \sec^3 x \, dx = \int \sec x \sec^2 x \, dx$ Use parts:

$u = \sec x \qquad dv = \sec^2 x \, dx$

$du = \sec x \tan x \, dx \qquad v = \tan x$

$\int \sec^3 x \, dx = \sec x \tan x - \int \sec x \tan^2 x \, dx$

$\tan^2 x = \sec^2 x - 1$

$= \sec x \tan x - \int \sec x (\sec^2 x - 1) \, dx$

$= \sec x \tan x - \int \sec^3 x \, dx - \int \sec x \, dx$

Add $\int \sec^3 x \, dx$ to both sides:

$$2\int \sec^3 x \, dx = \sec x \tan x - \int \sec x \, dx$$

$$= \sec x \tan x - \ln|\sec x + \tan x| + C$$

$$\int \sec^3 x \, dx = \frac{1}{2}\sec x \tan x - \frac{1}{2}\ln|\sec x + \tan x| + C$$

e) $\int \tan^2 x \, dx = \int \sec^2 x - 1 \, dx = \tan x - x + C$

f) $\int \tan^3 x \, dx = \int \tan x(\sec^2 x - 1) \, dx$

$= \int \tan x \sec^2 x \, dx - \int \tan x \, dx$

$= \frac{1}{2}\tan^2 x - \ln|\sec x| + C$

g) $\int \sin x \tan x \, dx = \int \frac{\sin^2 x}{\cos x} dx = \int \frac{1-\cos^2 x}{\cos x} dx$

$= \int \frac{1}{\cos x} - \cos x \, dx = \int \sec x - \cos x \, dx$

$= \ln|\sec x + \tan x| - \sin x + C$

h) $\int \sin^3 x \cos^2 x \, dx = \int \sin x (1 - \cos^2 x) \cos^2 x \, dx$

$= \int (\cos^2 x - \cos^4 x) \sin x \, dx$ Substitute $u = \cos x$

$= \int (u^2 - u^4)(-du) = -\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C$

$\sin x \cos x = \frac{1}{2}\sin 2x$

i) $\int \sin^2 x \cos^2 x \, dx = \frac{1}{4}\int \sin^2 2x \, dx$

$= \frac{1}{8}\int 1 - \cos 4x \, dx$ since $\sin^2 2x = \frac{1-\cos 4x}{2}$

$= \frac{1}{8}x - \frac{1}{32}\sin 4x + C$

j) $\int \sin 2x \cos 3x \, dx = \int \frac{1}{2}\left[\sin(2x-3x) + \sin(2x+3x)\right] dx$

$= \frac{1}{2}\int \sin(-x) + \sin 5x \, dx$

$= \frac{1}{2}\int -\sin x + \sin 5x \, dx$

$= \frac{1}{2}\cos x - \frac{1}{10}\cos 5x + C$