1. Plot some vectors in each quadrant for each vector field.

\[ a) \mathbf{F}(x, y) = \frac{y \mathbf{i} - x \mathbf{j}}{\sqrt{x^2 + y^2}} \quad b) \mathbf{F}(x, y) = x \mathbf{i} - y \mathbf{j} \]

\[ c) \mathbf{F}(x, y) = \nabla f(x, y) \quad f(x, y) = \ln \sqrt{x^2 + y^2} \]

2. Evaluate \( \int_C f(x, y) \, ds \).

\[ a) \quad f(x, y) = \sqrt{x^2 + y^2} \quad x = t \cos t, \quad y = t \sin t, \quad 0 \leq t \leq \pi \]

\[ b) \quad f(x, y) = e^{x+y} \quad C \text{ is the triangle with vertices } (0,0), (1,0) \text{ and } (0,1) \]

3. Evaluate each line integral.

\[ a) \int_C \sin y \, dy \quad C : y = x^2, \quad 0 \leq x \leq \sqrt{\pi} \]

\[ b) \int_C x \sqrt{y} \, dx + 2y \sqrt{x} \, dy \quad C : \quad y = \sqrt{1-x^2} \quad \text{from } (1,0) \text{ to } (0,1) \]

\[ c) \oint_C \mathbf{F} \cdot d\mathbf{r} \quad \mathbf{F} = \langle x^2, xy, z^2 \rangle \quad C : \quad \mathbf{r}(t) = \langle \sin t, \cos t, t^2 \rangle \quad 0 \leq t \leq \pi/2 \]

4. Find the work done if the force \( \mathbf{F}(x, y) = \langle x^2, xy \rangle \) is applied once counterclockwise

a) around the circle \( x^2 + y^2 = 4 \).

b) around the boundary of the triangle from (0,0) to (2,0) to (0,2) and back to (0,0)
5. Evaluate each line integral.

a) \[ \int_{C} \nabla f \cdot dr \] \quad f(x, y) = (x - y)e^{x^2+y^2} \quad C : x = 4 \cos t \quad y = 4 \sin t \quad 0 \leq t \leq \pi/2 \]

b) \[ \int_{C} \vec{F} \cdot dr \] \quad \vec{F}(x, y) = -y \sin xy \vec{i} + x \cos xy \vec{j} \quad C \text{ is the line segment from } (0,0) \text{ to } (1, \pi/3). \]

c) \[ \int_{C} \vec{F} \cdot dr \] \quad \vec{F}(x, y) = (-y \sin(xy) + y) \vec{i} - (x \sin(xy)) \vec{j} \quad C \text{ is the line segment from } (0,0) \text{ to } (1, \pi/3). \]

6. Find a potential function if possible or show the force function is not conservative.

a) \[ \vec{F}(x, y) = (ye^{xy} + 4x^3y) \vec{i} + (xe^{xy} + x^4) \vec{j} \]

b) \[ \vec{F}(x, y) = xye^{xy} \vec{i} + x^2e^{xy} \vec{j} \]

c) \[ \vec{F}(x, y) = (y \cos x - \cos y) \vec{i} + (\sin x + x \sin y) \vec{j} \]

d) \[ \vec{F}(x, y) = (e^{2x} + x \sin y) \vec{i} + x^2 \cos y \vec{j} \]

7. Show \( \vec{F} \) is independent of path and evaluate \[ \int_{C} \vec{F} \cdot dr \].

a) \[ \vec{F}(x, y, z) = 2x y^3 z^4 \vec{i} + 3x^2 y^2 z^4 \vec{j} + 4x^2 y^3 z^3 \vec{k} \] \quad C : \quad \{t, t^2, t^3\} \quad 0 \leq t \leq 2 \]

b) \[ \vec{F}(x, y) = (2y^2 - 12x^3y^3 + 3x^2) \vec{i} + (4xy - 9x^4y^2 + 4y^3) \vec{j} \] \quad C \text{ is the line segment from } (0,0) \text{ to } (2,3) \]

c) \[ \vec{F}(x, y) = -\frac{y}{x^2} \vec{i} + \frac{1}{x} \vec{j} \] \quad C \text{ is any path, not crossing the y-axis, from } (1,1) \text{ to } (2,6). \text{ Does the Fundamental Theorem apply for the line segment from } (-2,3) \text{ to } (2,5)?
8. Evaluate

\[ \int_C \mathbf{F} \cdot d\mathbf{r} \quad \mathbf{F} = ye^{x^2+y^2} \mathbf{i} + xe^{x^2+y^2} \mathbf{j} \]  
\( C \) is the line segment from (0,0) to (1,0) followed by the quarter of the unit circle from (1,0) to (0,1) followed by the line segment from (0,1) to (0,0).

9. Evaluate

\[ \int_C (3x^2y^4 - xy) \, dx + 4x^3y^3 \, dy \]  
\( C \) is the boundary of the quarter circle of radius 1 from (1,0) to (0,1). Note that \( C \) is not closed but the integrals on the line segments (0,1) to (0,0) and (0,0) to (1,0) are 0.

10. Evaluate using Green’s Theorem.

a) \[ \int_C \mathbf{F} \cdot d\mathbf{r} \quad \mathbf{F} = ye^{x^2+y^2} \mathbf{i} + xe^{x^2+y^2} \mathbf{j} \]  
\( C \) is the boundary of the sector of the unit circle \( 0 \leq \theta \leq \frac{\pi}{4}, \quad 0 \leq r \leq 1. \)

b) \[ \int_C (y^2 - \arctan x) \, dx + (3x + \sin y) \, dy \]  
\( C \) is the curve \( y = x^2 \) from (-2,4) to (2,4) followed by the line segment from (2,4) to (-2,4).

c) \[ \int_C \mathbf{F} \cdot d\mathbf{r} \quad \mathbf{F} = xy^2 \mathbf{i} + y \arcsin x \mathbf{j} \]  
\( C \) is the boundary of the quarter of the unit circle from (1,0) to (0,1). Note: \( C \) is not closed.