1. Sketch a contour map for each function.

   \( a) \ f(x, y) = \frac{x^2}{4} + \frac{y^2}{9} \) \quad \( b) \ f(x, y) = e^{xy} \)

2. Describe the level surfaces for \( w = f(x, y, z) = x^2 - y^2 + 4z^2 \) for \( w = -1, w = 0, w = 1 \).

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3. Find any points of discontinuity of the function.

   \( a) \ f(x, y) = \begin{cases} \frac{xy}{x^2 + xy + y^2} & (x, y) \neq (0,0) \\ 0 & (x, y) = (0,0) \end{cases} \)

   \( b) \ f(x, y) = \begin{cases} \frac{x^2y^3}{2x^2 + y^2} & (x, y) \neq (0,0) \\ 1 & (x, y) = (0,0) \end{cases} \)

4. Evaluate each limit.

   \( a) \ \lim_{(x, y) \to (0, 0)} \frac{xy + 1}{x^2 + y^2 + 1} \quad b) \ \lim_{(x, y) \to (0, 1)} \frac{xy - x}{x^2 + y^2 - 2y + 1} \)

5. Find all first partial derivatives of each function.

   \( a) \ f(x, y) = \frac{x^3 + y^3}{x^2 + y^2} \quad b) \ f(x, y) = \ln(1 + xy) \)

   \( c) \ f(x, y, z) = x^2e^{xy} + ze^{xz} \quad d) \ f(x, y, z) = \sqrt{x^2y^3z^3} \)

   \( e) \ z = f(x, y) \quad and \quad xy + yz = xz \)
6. Find all 2nd partial derivatives of the function. Verify Clairaut’s Theorem in each case.

\( a) \quad f(x, y) = x^2 e^{xy} \quad b) \quad f(x, y) = x^3 y^4 - 2x^2 y \)

7. Verify that the Laplace equation is true for \( f(x, y) = \ln \sqrt{x^2 + y^2} \)

8. Is there a function with continuous 2nd partial derivatives

for which \( f_x(x, y) = x^2 e^{xy} \quad \text{and} \quad f_y(x, y) = y^2 e^{xy} \)?

9. Find an equation of the tangent plane to the graph of the given function at the given point.

\( a) \quad f(x, y) = x^3 - 2xy + y^3 \quad \text{at} \quad P(2, -1, 1) \)
\( b) \quad f(x, y) = e^{xy} \quad \text{at} \quad P(0, 2, 1) \)
\( c) \quad f(x, y) = xe^{x^2 y} \quad \text{at} \quad P(1, 0, 1) \)

10. Find the total differential of each function.

\( a) \quad z = f(x, y) = e^y \sin(xy) \quad b) \quad w = f(x, y, z) = e^{x^2 y^3 z} \)
\( c) \quad z = f(x, y) = \ln \left[ 1 + (x - y)^2 \right] \)

11. #30 in Stewart

The dimensions of a closed rectangular box are measured as 80cm, 60cm and 50cm, with a possible error of 0.2cm in each dimension. Use differentials to estimate the maximum error in calculating the surface area of the box.

12. #32 in Stewart

Use differentials to estimate the amount of metal in a closed cylindrical can that is 10cm high and 4cm in diameter if the metal in the wall is 0.05 cm thick and metal in the top and bottom is 0.1 cm thick.