1. Find the work done if the force $F(x, y) = x^2 \vec{i} + xy \vec{j}$ is applied along the curve $y = \sqrt{x}$ from the point (0,0) to (1,1) followed by the line segment from (1,1) to (2,4).

2. Find the mass of a wire in the shape of the curve $C$: $y = x^3$ from (0,0) to (2,8) with density function, $\rho(x, y) = y$.

3. Find the work done by the force $F(x, y) = x^2 \vec{i} + xy \vec{j}$ applied once around the semicircle $y = \sqrt{4-x^2}$ followed by the line segment from (-2,0) to (2,0).

4. Find the mass of a wire in the shape of the quarter circle of radius 3 centered at (0,0) from (3,0) to (0,3), if the density is given by $\rho(x, y) = x + y$.

5. Show that the vector force, $F(x, y) = (ye^{xy} + 4x^3y) \vec{i} + (xe^{xy} + x^4) \vec{j}$, is conservative by finding a potential function, and evaluate the work done in applying this force along any curve from (1,0) to (2,3).

6. Evaluate $\int_C x^2 \, dy + xy \, dx$ where $C$ is the perimeter of the rectangle with corners (-1,-1), (-1,1), (1,1), and (1,-1).

7. a) Evaluate $\int_C x^3 \, dy + x^4 \, dx$ where $C$ is the quarter circle, $x^2 + y^2 = 1$ in the first quadrant.

b) Evaluate $\int_C x^3 \, dx + x^4 \, dy$ where $C$ is as in a.

c) Since $C$ is not closed, we cannot use Green’s theorem unless we close it with the line segments from (0,1) to (0,0) and on to (1,0). Use Green’s theorem for the closed curve and find the answers to a and b by subtracting the integrals along the line segments in each case.

8. a) Is there a vector field, $F$ on $\mathbb{R}^3$ for which $\text{curl}(F) = x^2 \vec{i} + y^2 \vec{j}$?

b) Is the vector field $\vec{F} = yz \vec{i} - z^2 \vec{j} + x^2 \vec{k}$ conservative?
9. Find the surface area of the part of the hyperbolic paraboloid \( z = y^2 - x^2 \) that lies between the cylinders \( x^2 + y^2 = 1 \) and \( x^2 + y^2 = 4 \).

10. Find the tangent plane to the surface parameterized by \( \mathbf{r}(u,v) = uv \mathbf{i} + ue^v \mathbf{j} + ve^u \mathbf{k} \) at the point \((2,2e,e^2)\).

11. Find the flux of \( \mathbf{F} = e^y \mathbf{i} + ye^x \mathbf{j} + x^2 y \mathbf{k} \) across the surface \( S: z = x^2 + y^2 \) above \( 0 \leq x \leq 1, \ 0 \leq y \leq 1 \).

12. \( S \) is the surface of the paraboloid \( z = x^2 + y^2 \) for \( z \leq 4 \).
   
   a) Parametrize \( S \) using a polar form and find the surface area.
   
   b) Evaluate \( \iint_S \frac{xyz}{(x^2 + y^2)^2} \, dS \).

13. Evaluate \( \iint_S (y^2 + z^2) \, dS \) where \( S: x = 4 - y^2 - z^2 \) in front of \( x = 0 \).

14. \( S \) is the cap of the sphere \( x^2 + y^2 + z^2 = 36 \) for \( z \geq 3 \).
   
   a) Parameterize \( S \) using a spherical form.
   
   b) Find the surface area of \( S \).
   
   c) Evaluate \( \iint_S x^2 z \, dS \).
15. Use Stokes’ theorem to find $\int_C F \cdot dr$.

\[ F = 2z \, i + 4x \, j + 5y \, k \quad \text{C is the intersection of} \quad z = x + 4 \quad \text{and} \quad x^2 + y^2 = 4. \]

\[ F = 4y \, i + 6x \, j - 2z \, k \quad \text{C is the intersection of} \quad z = 9 - 2x \quad \text{and} \quad z = 9 - x^2 - y^2. \]

\[ F = 2yz \, i - xy \, j + x^2 \, k \quad \text{C is the intersection of} \quad z = 4 - (x - 2)^2 - y^2 \quad \text{and} \quad z = x^2 + y^2. \]