Section 13.2  Iterated Integrals over rectangles and Fubini's Theorem

Fubini's theorem for a continuous function $f(x,y)$
If $R$ is the rectangle $[a, b] \times [c, d]$ then by Fubini's theorem
\[
\int_A f(x,y) \, dA = \int_c^d \int_a^b f(x,y) \, dx \, dy = \int_a^b \int_c^d f(x,y) \, dy \, dx
\]

Examples:
1. Evaluate $\int_0^2 \int_0^3 (x^2 + xy) \, dy \, dx$. Reverse the order of integration and verify Fubini's them in this case.
2. Find the volume of the solid with height $f(x, y) = x^3 y$ and base $[1,5] \times [2,4]$.

Section 13.3 Iterated Integrals over general regions.
Examples.
1. Evaluate $\int_R x^2 \, dA$ where $R$ is the region bounded by $x = 0$, $y = 1$, and $y = x$.
2. For the region of problem 1, set up $\int_R xe^y \, dA$ two ways and evaluate one setup.
3. Evaluate $\int_0^1 \int_0^1 e^{x^3} \, dxdy$ by reversing the order of integration.
4. Problem 20 in Stewart Find the volume under the paraboloid $z = 3x^2 + y^2$ over the region bounded by $y = x$ and $x = y^2 - y$.
5. Problem 24 in Stewart Find the volume bounded by the cylinder $y^2 + z^2 = 4$ and the planes $x=2y$, $x=0$, and $z=0$ in the first octant.