If three masses \( m_1, m_2, m_3 \) are located at the points \( (x_i, y_i) \quad i = 1, 2, 3 \) then the center of mass is at \((x, y)\) where \( x \) the weighted average of the \( x \) - values and \( y \) is the weighted average of the \( y \) - values.

\[
x = \frac{1}{M} \sum_{i=1}^{3} x_i m_i, \quad y = \frac{1}{M} \sum_{i=1}^{3} y_i m_i, \quad M = m_1 + m_2 + m_3.
\]

In general, for a lamina in the shape of region \( D \) with continuous density function \( \rho(x, y) \),

the center of mass is \((x, y)\) with

\[
x = \frac{1}{M} \iint_D x \rho(x, y) \, dA, \quad y = \frac{1}{M} \iint_D y \rho(x, y) \, dA, \quad M = \iint_D \rho(x, y) \, dA
\]

**Examples**

Find the center of mass for the lamina described by the region \( D \) with density function \( \rho(x, y) \).

1. \( D \) is the rectangle \((0,0)\) and \((2,3)\) as opposite corners, \( \rho(x, y) = y \).

2. \( D \) is the triangle with corners \((0,0)\), \((1,1)\) and \((4,0)\), \( \rho(x, y) = x \).

3. \( D \) is the unit disk \( x^2 + y^2 \leq 1 \), \( \rho(x, y) = K(x^2 + y^2) \), \( K \) a constant

4. Find the moment of inertia about the \( x \)-axis which is \( I_x = \iint_D y^2 \rho(x, y) \, dA \) for

\( D \) bounded by \( y = \sin x \), \( y = 0 \), \( 0 \leq x \leq \pi \) and constant density function, \( \rho \).