151 Week in Review
5.2, 5.3, 5.5 Solutions

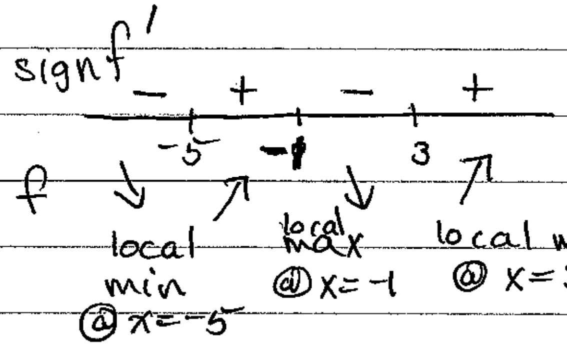
1. $f(x) = (2x^2+4x-30)^{2/5}$ Domain $f = (-\infty, \infty)$

$$f'(x) = \frac{2}{5}(2x^2+4x-30)^{-3/5}(4x+4)$$
$$= \frac{2(4x+4)}{5(2x^2+4x-30)^{3/5}}$$

a) Critical values are $x=-1$, $f'(-1)=0$ and the roots of $2x^2+4x-30$, where f' DNE.

$$2x^2+4x-30 = 2(x^2+2x-15)$$
$$= 2(x+5)(x-3) = 0 \text{ at } x=-5, x=3.$$

b)



$f'(0) < 0$, all exponents are odd so f' changes sign at each c.v.

local minimums are $(-5, 0)$ and $(3, 0)$
local maximum is $(-1, 4)$

c) Absolute max and min on $[-2, 4]$

x	$f(x)$	
-2	$(-30)^{2/5}$	← absolute min is $(-30)^{2/5}$ at $x=-2$
-1	4	← absolute max is 4 at $x=-1$
3	0	
4	$(18)^{2/5} < 4$	

1d)

x	$f(x)$
-2	$(-30)^{2/5}$
-1	4
3	0
5	$(40)^{2/5} > 4$

Abs. min is $(-30)^{2/5}$ at $x=-2$

Abs. max is $(40)^{2/5}$ at $x=5$

2. $g(x) = e^x f(x)$ $f(0)=2$ $f'(0)=-2$ $f''(0)=1$

Using the product rule:

$g'(x) = e^x f(x) + e^x f'(x)$

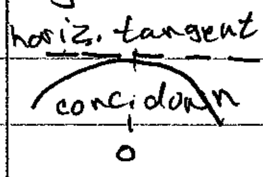
The 2nd derivative test only applies if $g'(0)=0$.

$g'(0) = e^0 f(0) + e^0 f'(0) = 0$ ✓

$$g''(x) = [e^x f(x) + e^x f'(x)] + [e^x f'(x) + e^x f''(x)]$$

$$= e^x f(x) + 2e^x f'(x) + e^x f''(x).$$

$g''(0) = f(0) + 2f'(0) + f''(0) = 2-4+1 < 0$



g has a local max at $x=0$.

3. $f(x) = \ln(x+3) + \frac{2}{(x+3)^2}$ Domain f $= (-3, \infty)$

$$f'(x) = \frac{1}{(x+3)} - \frac{4}{(x+3)^3} = \frac{x^2+6x+5}{(x+3)^3}$$

The critical values are the roots of x^2+6x+5;
-3 is not in Domain f.

$(x+5)(x+1) = 0$ at -5 and -1.
The only c.v. is at $x = -1$.

sign f': -3 ⋮ $-$ $|_{-1}$ $+$ local min at $(-1, \frac{1}{2} + \ln 2)$

f ⋮ ↓ ↗

$$f''(x) = \left[(2x+6)(x+3)^3 - (x^2+6x+5)(3(x+3)^2)\right] \Big/ (x+3)^6$$

$$= \frac{(x+3)^2}{(x+3)^6}\left[(2x+6)(x+3) - 3(x^2+6x+5)\right]$$

$$= \frac{-1}{(x+3)^4}\left[x^2+6x-3\right]$$

roots are $\boxed{-3+2\sqrt{3}}$, $-3-2\sqrt{3}$ (not in Domain f)

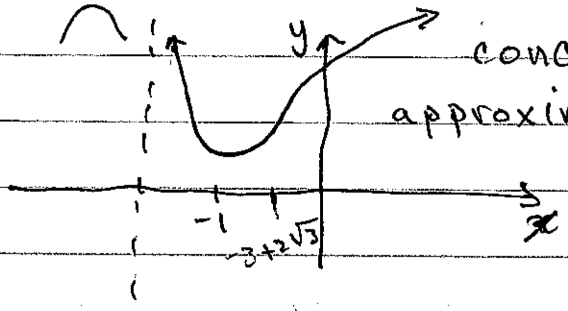
sign f'': $+$ $|_{-3+2\sqrt{3}}$ $-$

f ∪ ∩

inflection pt at $x = -3+2\sqrt{3}$

concave up on $(-3, -3+2\sqrt{3})$

approximate shape



4. $f(x) = x^3 + 3x^2 - 9x - 3$

$f'(x) = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3) = 3(x+3)(x-1)$

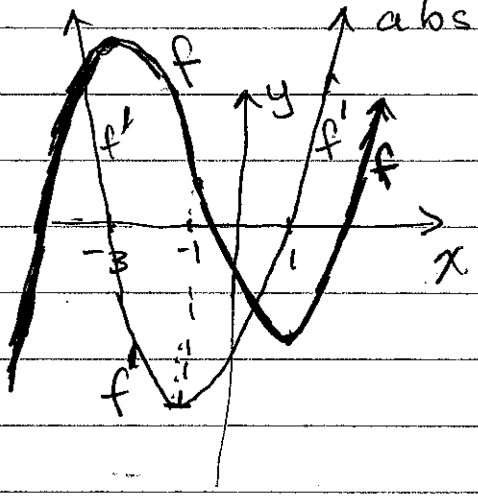
c.v. are $x = -3$ and $x = 1$.

a)

x	$f(x)$	
-6	-57	← absolute min. of -57 at $x=-6$
-3	24	← absolute max of 24 at $x=-3$
1	-8	
2	-1	

b) on $[-3, 1]$, abs. max is 24 at $x = -3$

abs min is -8 at $x = 1$.



where f' has an extremum, f has an inflection.

5 a) Solve $f'(c) = \dfrac{f(2) - f(-1)}{2 - (-1)}$ $c \in [-1, 2]$

$f'(x) = 2x + 2$ $\qquad \dfrac{f(2) - f(-1)}{3} = 3$

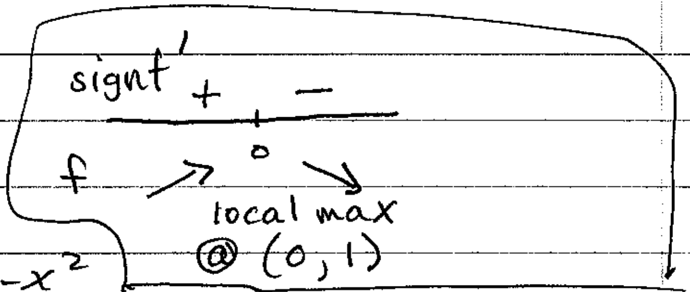
$f'(c) = 2c + 2$

$2c + 2 = 3$ $\qquad \boxed{c = \tfrac{1}{2}}$

b) $\dfrac{b^2 - a^2}{b - a} = 2c$ $\qquad \boxed{c = \dfrac{b+a}{2}}$, the midpoint

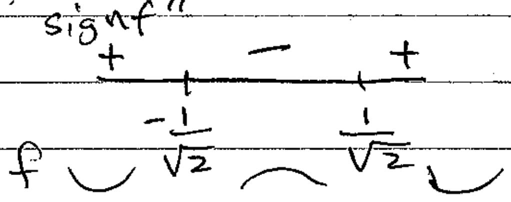
6. $f(x) = e^{-x^2}$

$f'(x) = -2x e^{-x^2}$



$f''(x) = -2e^{-x^2} + 4x^2 e^{-x^2}$

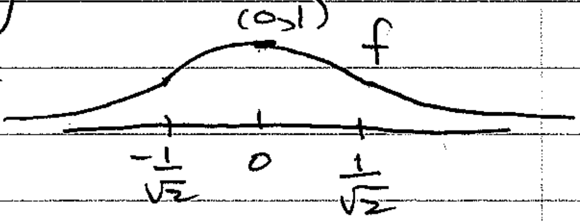
$= 2e^{-x^2}(2x^2-1)$



inflections at $\left(-\frac{1}{\sqrt{2}}, e^{-1/2}\right)$

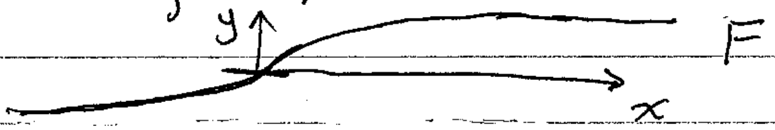
and $\left(\frac{1}{\sqrt{2}}, e^{-1/2}\right)$

$\lim_{x\to\infty} e^{-x^2} = 0 = \lim_{x\to-\infty} e^{-x^2}$



If $F'(x) = f(x)$ then $F' > 0$ for all x so F is always increasing.

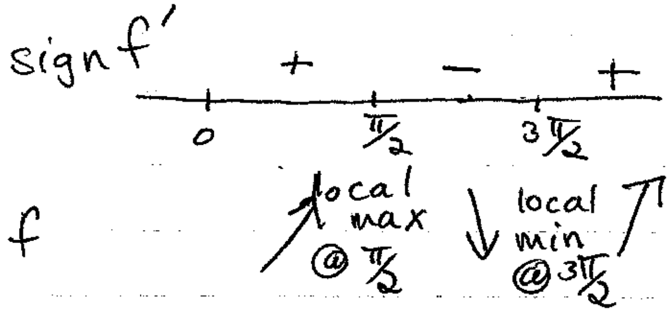
F has an inflection pt where $F' = f$ has an extremum, $(0,1)$.



7. $f(x) = x \sin x + \cos x$ on $[0, 2\pi]$

$f'(x) = \sin x + x\cos x - \sin x = x \cos x$

$f' = 0$ @ $x = 0, \frac{\pi}{2}, \frac{3\pi}{2}$



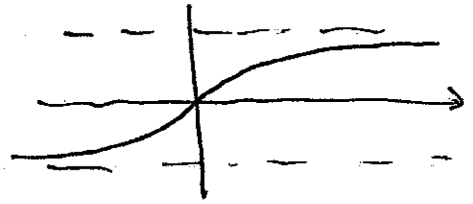
f is increasing on $(0, \frac{\pi}{2})$ and $(\frac{3\pi}{2}, 2\pi)$.

decreasing on $(\frac{\pi}{2}, \frac{3\pi}{2})$.

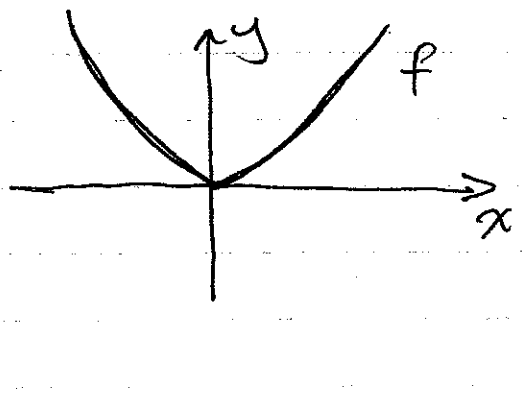
local max is @ $(\frac{\pi}{2}, \frac{\pi}{2})$

local min is @ $(\frac{3\pi}{2}, -\frac{3\pi}{2})$

8. $f'(x) = \arctan x + \frac{x}{1+x^2} - \frac{1}{2}\frac{2x}{1+x^2} = \arctan x$

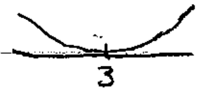


f' is always increasing so f is always concave up. f has a local min at (0,0)



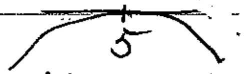
9. At $x=1$, the 2nd derivative test does not apply since $f'(1) \neq 0$.
We know f is increasing, concave up.

At $x=2$, no conclusion can be made.

At $x=3$  f has a local min.

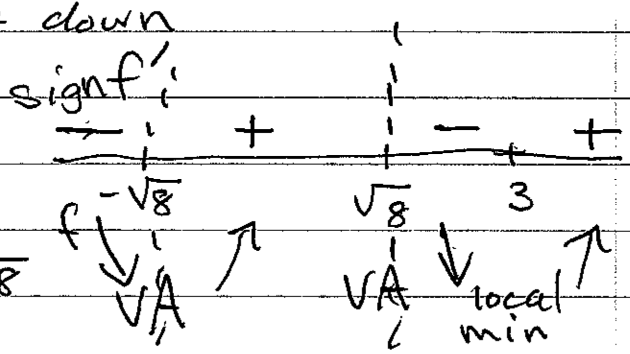
(Horiz tangent & concave up)

At $x=4$, it does not apply. f is increasing concave down.

At $x=5$,  f has a local max.

Horiz. tangent & concave down

10. $f'(x) = \dfrac{x-3}{x^2-8}$



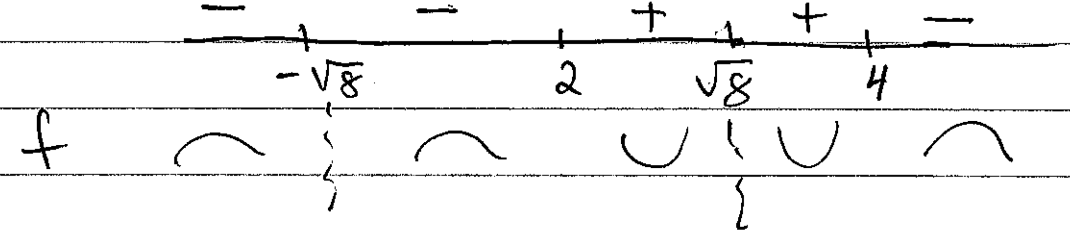
critical values are $x=3$, $x=-\sqrt{8}$, $x=\sqrt{8}$

f has a local min @ $x=3$. $x=-\sqrt{8}$, $x=\sqrt{8}$ are given as vertical asymptotes.
f is decreasing on $(-\infty, -\sqrt{8})$ and on $(\sqrt{8}, 3)$.
f is increasing on $(-\sqrt{8}, \sqrt{8})$ and $(3, \infty)$.

$$f''(x) = \frac{x^2-8-(x-3)\cdot 2x}{(x^2-8)^2} = \frac{-(x^2-6x+8)}{(x^2-8)^2}$$

$$= -(x-4)(x-2)\Big/(x^2-8)^2$$

sign f''



f has inflection pts. @ $x=2$ and $x=4$.
f is concave down on
$(-\infty, -\sqrt{8})$, $(-\sqrt{8}, 2)$ and $(4, \infty)$

f is concave up on $(2, \sqrt{8})$ and $(\sqrt{8}, 4)$

11. Surface Area $= \underset{\text{top + bottom}}{2\pi r^2} + \underset{\text{lateral surface}}{2\pi r h}$

$$V = \pi r^2 h = 16\pi$$

$$h = \frac{16}{r^2}$$

$$S(r) = 2\pi r^2 + 2\pi r \frac{16}{r^2} = 2\pi r^2 + \frac{32\pi}{r}$$

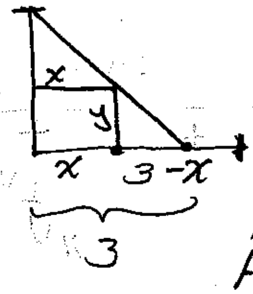
$$S'(r) = 4\pi r - \frac{32\pi}{r^2} = 0 \text{ if}$$

$$4r^3 = 32 \qquad r = 2$$

$$S''(r) = 4\pi + \frac{64\pi}{r^3} > 0 \text{ when } r = 2$$

so we have a min. at $r=2$, $h=4$

12.



$A = xy$ $\quad \frac{y}{3-x} = \frac{4}{3}$ $\quad y = 4 - \frac{4}{3}x$

$A = 4x - \frac{4}{3}x^2$ $\quad A'(x) = 4 - 8/3\,x$

$A' = 0$ if $x = 1.5$ and then $y = 2$.

$A_{max} = 3$ sq units. ($A'' < 0$ so it's a max)