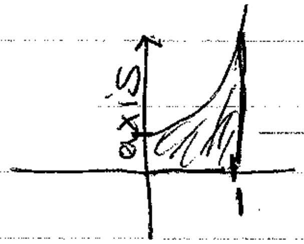
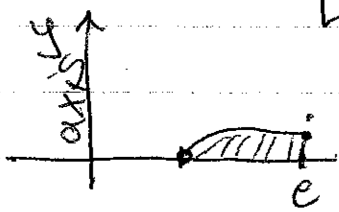
Math 152 WIR
7.4, 7.5 Solutions

1. 

$$2\pi\int_0^1 x e^{x^2}\,dx = 2\pi\left[\tfrac{1}{2}e^{x^2}\Big|_0^1\right]$$

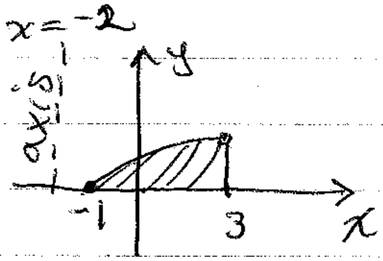
$$= \boxed{\pi(e-1)}$$

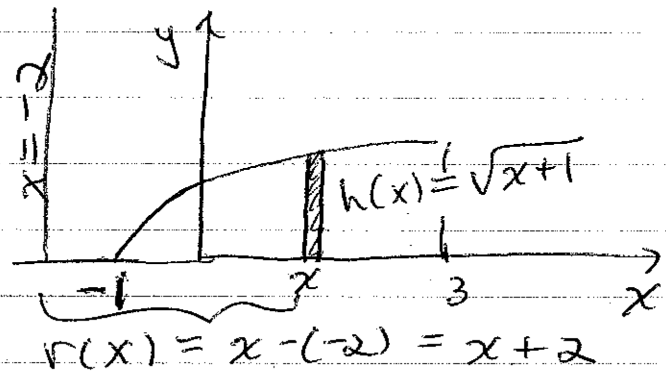
2. 

$$2\pi\int_1^e x\,\frac{\ln x}{x^2}\,dx = 2\pi\int_1^e \frac{\ln x}{x}\,dx$$

$$= 2\pi\left[\tfrac{1}{2}(\ln x)^2\Big|_1^e\right] = 2\pi\left(\tfrac{1}{2}\right)(1-0) =$$

$$= \boxed{\pi}$$

3. 



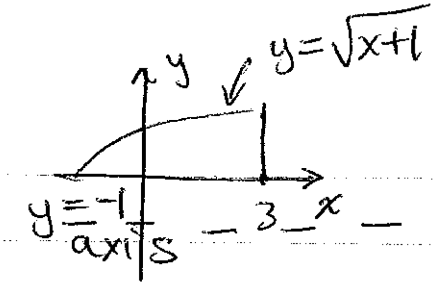
$$2\pi\int_{-1}^{3}(x+2)\sqrt{x+1}\,dx \qquad u = x+1 \quad x = u-1$$
$$du = dx \quad x+2 = u+1$$

$$2\pi\int_0^4 (u+1)\sqrt{u}\,du = 2\pi\int_0^4 u^{3/2} + u^{1/2}\,du$$

$$= 2\pi\left[\tfrac{2}{5}u^{5/2} + \tfrac{2}{3}u^{3/2}\Big|_0^4\right] = 2\pi\left[\tfrac{64}{5} + \tfrac{16}{3}\right]$$

$$= \boxed{544\pi}$$

4.



Using washers: $R = \sqrt{x+1} + 1$
$r = 1$

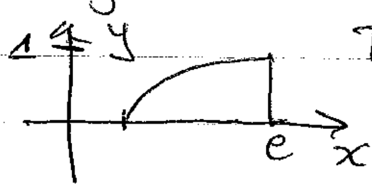
$$\pi \int_{-1}^{3} (\sqrt{x+1} + 1)^2 - 1 \; dx$$

$$= \pi \int_{-1}^{3} (x+1) + 2\sqrt{x+1} + \cancel{1} - \cancel{1} \; dx$$

$$= \pi \left[\tfrac{1}{2}(x+1)^2 + \tfrac{4}{3}(x+1)^{3/2} \Big|_{-1}^{3} \right]$$

$$= \pi \left[2 + \tfrac{32}{3} - (0) \right] = \boxed{\frac{38\pi}{3}}$$

5. $y^2 = \ln x \quad 1 \le x \le e \qquad y = \sqrt{\ln x}$
$y \ge 0$



$\pi \int_{1}^{e} \ln x \; dx$ Using washers.

Using shells: $x = e^{y^2} \quad h(y) = e - e^{y^2}$
$r(y) = y$

$$2\pi \int_{0}^{1} y(e - e^{y^2})\,dy = 2\pi \left[\tfrac{1}{2} e y^2 - \tfrac{1}{2} e^{y^2} \right]_{0}^{1}$$

$$= \boxed{\pi}$$

6. $W = \int_{0.5}^{1} \sin\left(\frac{\pi}{2}x\right) dx = -\frac{2}{\pi}\cos\left(\frac{\pi}{2}x\right)\Big|_{0.5}^{1}$

$= 0 - \left(-\frac{2}{\pi}\cos\frac{\pi}{4}\right) = \frac{2}{\pi}\frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{2}}{\pi} J}$

$6 \text{ in} = \frac{1}{2} \text{ ft.}$

7. a) $\frac{1}{2}k = 4 \quad k = 8 \quad W = \int_0^2 8x\,dx = 4x^2\Big|_0^2 = 16$ ft-lbs

b) $\int_1^2 8x\,dx = 4x^2\Big|_1^2 = 16 - 4 = 12$ ft-lbs

8. $\int_{1-L}^{1.5-L} kx\,dx = 6 \qquad \int_{1.5-L}^{2-L} kx\,dx = 10$

$\frac{1}{2}k\left[(1.5-L)^2 - (1-L)^2\right] = 6 \qquad \frac{1}{2}k\left[(2-L)^2 - (1.5-L)^2\right] = 10$

$k\left[1.25 - L\right] = 12 \qquad k\left[1.75 - L\right] = 20$

subtracting the 2nd equation from the 1st we have

$.5k = 8$ so $k = 16$ Subctituting $k=16$

we have $1.75 - L = \frac{20}{16} = 5/4 = 1.25$

so $\boxed{L = .5}$

9. Since the cable is uniform it weighs $\frac{60}{40}$ lbs/ft. or 1.5 lbs/ft.

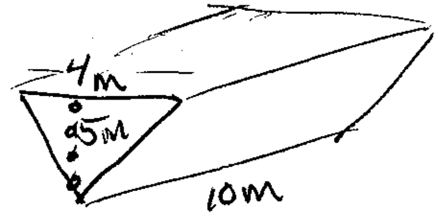
The lower 30 ft. of cable is pulled up 10 ft. and it weighs 45 lbs. The work for this portion is 450 ft-lbs.

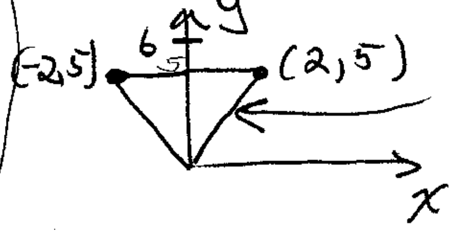
The increment y ft from the top is pulled up y ft, if $0 < y \le 10$. This increment weighs $1.5dy$ lbs. The work done is $1.5y\,dy$ so the work for the top ~~10 ft~~ of cable is $\int_0^{10} 1.5y\,dy = .75y^2\Big|_0^{10} = 75$ ft-lbs

$W = 75 + 450 = 525$ ft-lbs

10.

a)





$y = \frac{5}{2}x \quad x = \frac{2}{5}y$

$2x = \frac{4}{5}y$

width at ht. y

Measuring from the bottom of the tank, the incremental slice at height y weighs $\rho g \times 2x\,dy = \rho g \frac{4}{5} y\,dy$

This slice of water is lifted from height y to height 6m so it is lifted $6-y$ meters.

$$dW = \rho g \frac{4}{5} y(6-y)\,dy$$

We have slices from $y=0$ to $y=5$ so

$$W = \rho g \int_0^5 \frac{4}{5} y(6-y)\,dy = \frac{80}{3}\rho g \text{ J}$$

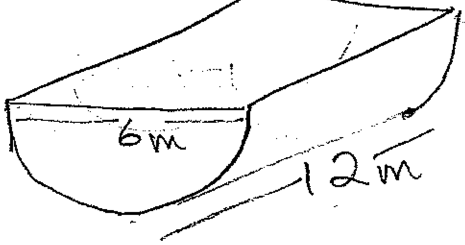
$$= \frac{80}{3}(9800) \text{ J}$$

$$= 261333\tfrac{1}{3} \text{ J}$$

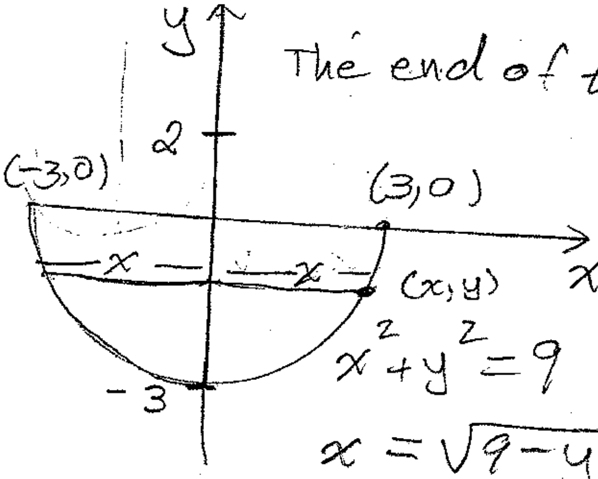
b) The set-up is the same except we have slices only from $y=0$ to $y=3$.

$$W = \rho g \int_0^3 \frac{4}{5} y(6-y)\,dy = \frac{216}{15}\rho g = 141120 \text{ J}$$

11.



The end of the tank



$$x^2+y^2=9$$

$$x=\sqrt{9-y^2}$$

$\boxed{2x = 2\sqrt{9-y^2} \text{ is the width of the slice at } y.}$

The weight of the slice at y is

$$\rho g\, 2\sqrt{9-y^2}\,(12)\,dy = 24\rho g\sqrt{9-y^2}\,dy$$

This must be lifted from level y up to level 2 so $2-y$ meters.

$$\text{Work} = 24\rho g\int_{-3}^{0}(2-y)\sqrt{9-y^2}\,dy$$

Distributing we have:

$$48\rho g\int_{-3}^{0}\sqrt{9-y^2}\,dy - 24\rho g\int_{-3}^{0}y\sqrt{9-y^2}\,dy$$

The first integral is $48\rho g$ times $\frac{1}{4}$ the area of the circle of radius 3. The second can be done by substituting

$$u = 9-y^2$$
$$du = -2y\,dy$$
$$-\tfrac{1}{2}du = y\,dy$$

The 2nd integral:

$$\int_{-3}^{0} y\sqrt{9-y^2}\,dy = -\frac{1}{2}\int_{0}^{9} \sqrt{u}\,du$$

$$= -\frac{1}{2}\left(\frac{2}{3}\right)u^{3/2}\Big|_0^9 = -9$$

$$\text{Work} = 48\rho g\left(\frac{9\pi}{4}\right) + 24\rho g\,(9)$$

$$= 108\pi\rho g + 216\rho g$$