Solutions WIR 151 1.3 – 2.3

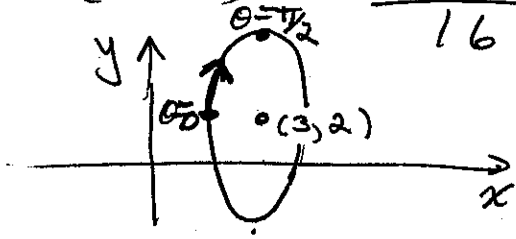
1. $x = 2t+1 \qquad \frac{x-1}{2} = t \Rightarrow y = \sqrt{4-t} = \sqrt{4 - \frac{x-1}{2}}$

$$y = \sqrt{\frac{9-x}{2}}$$

2. $x-3 = -\cos\theta \qquad \frac{y-2}{4} = \sin\theta \qquad \cos^2\theta + \sin^2\theta = 1$

so $(x-3)^2 + \left(\frac{y-2}{4}\right)^2 = 1$

$(x-3)^2 + \frac{(y-2)^2}{16} = 1$ describes an ellipse



3. Find $\vec{v}$ which is parallel to the line:

$\vec{v} = \overrightarrow{P(3,1)Q(7,4)} = \langle 4, 3\rangle$

$\vec{r}(t) = \overrightarrow{OP} + \vec{v}\, t = \langle 3,1\rangle + \langle 4, 3\rangle t$

$= (3+4t)\vec{i} + (1+3t)\vec{j}$

Parametric equations: $x = 3+4t \quad y = 1+3t$

4. $x = 2 - t \quad y = 1 + 2t$ This line is parallel to $\vec{v}_1 = \langle -1, 2 \rangle$.

$\vec{r} = (5 + 6s)\vec{i} + (-1 + 3s)\vec{j}$ is parallel to $\vec{v}_2 = \langle 6, 3 \rangle$

$\vec{v}_1 \cdot \vec{v} = -6 + 6 = 0$ so the lines are perpendicular,

The lines intersect when $x(t) = 2 - t = x(s) = 5 + 6s$
and $y(t) = 1 + 2t = y(s) = -1 + 3s$

Solve

$$2 - t = 5 + 6s$$
$$1 + 2t = -1 + 3s$$

Multiply the 1st eqn. by 2 and add to the 2nd so t drops out:

$$\begin{array}{rcl} 4 - 2t &=& 10 + 12s \quad \text{add} \\ 1 + 2t &=& -1 + 3s \\ \hline 5 &=& 9 + 15s \end{array}$$

$$-4 = 15s$$
$$s = -\frac{4}{15}$$

Solve for $x(s)$ and $y(s)$:

$x = 5 + 6\left(\frac{-4}{15}\right) \qquad y = -1 + 3\left(\frac{-4}{15}\right)$

$x = \frac{17}{5} = 3.4 \qquad y = -\frac{9}{5} = -1.8$

5. $\lim\limits_{x \to 3} \dfrac{x^2 - 7x + 12}{x^2 - 5x + 6}$

Substiting $x = 3$ gives "$\frac{0}{0}$" which is indeterminate.

Factor each: $\dfrac{(x-3)(x-4)}{(x-3)(x-2)} = \dfrac{x-4}{x-2}$ if $x \neq 3$.

$$\lim_{x \to 3} \frac{x^2 - 7x + 12}{x^2 - 5x + 6} = \lim_{x \to 3} \frac{x-4}{x-2} = -1$$

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6. See transparency

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7. $\lim\limits_{t \to 3} \left[\dfrac{t+3}{2t} \vec{i} + \dfrac{t^2 - 4t + 3}{t^2 + t - 12} \vec{j} \right]$

$x(t) \to \frac{6}{6} = 1$

Subst. into $y(t)$ gives $\dfrac{9 - 12 + 3}{9 + 3 - 12} = \frac{0}{0}$ is indet.

Reduce: $\dfrac{\cancel{(t-3)}(t-1)}{(t+4)\cancel{(t-3)}}$ $t \neq 3$

$= \dfrac{t-1}{t+4} \xrightarrow[t \to 3]{} \frac{2}{7}$

So The limit is

$\vec{i} + \frac{2}{7} \vec{j}$

8 a) $\lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25}$ Substituting gives "$\frac{0}{0}$". Do algebra.

$$(x - 25) = (\sqrt{x} - 5)(\sqrt{x} + 5)$$

$$\frac{\sqrt{x} - 5}{x - 25} = \frac{\sqrt{x} - 5}{(\sqrt{x} - 5)(\sqrt{x} + 5)} = \frac{1}{\sqrt{x} + 5} \text{ if } x \neq 25$$

$$\lim_{x \to 25} \frac{1}{\sqrt{x} + 5} = \frac{1}{10}$$

b) $\lim_{x \to 2^+} \frac{\sqrt{x^2 - 4}}{x - 2}$ Substituting gives "$\frac{0}{0}$"

$$x^2 - 4 = (x - 2)(x + 2)$$

$$\frac{\sqrt{x^2 - 4}}{x - 2} = \frac{\sqrt{x - 2}\,\sqrt{x + 2}}{x - 2} = \frac{\sqrt{x + 2}}{\sqrt{x - 2}} \text{ if } x \neq 2.$$

$$\lim_{x \to 2^+} \frac{\sqrt{x^2 - 4}}{x - 2} = \lim_{x \to 2^+} \frac{\sqrt{x + 2}}{\sqrt{x - 2}}$$

Substituting gives "$\frac{4}{0}$" so the limit is $+\infty$, $-\infty$ or DNE.

Since $\frac{\sqrt{x+2}}{\sqrt{x-2}} > 0$, the limit is $+\infty$.

Subst. gives $\frac{0}{0}$, indeterminate

8c) One way Subst. $k = h+4$ Then $k \to 9$, $h \to 5$

$$\lim_{k\to 9} \frac{k-9}{3-\sqrt{k}} = \lim_{k\to 9} \frac{\cancel{(\sqrt{k}-3)}(\sqrt{k}+3)}{-\cancel{(\sqrt{k}-3)}}$$

$$= \lim_{k\to 9} -(\sqrt{k}+3) = -6$$

OR

$$\frac{h-5}{3-\sqrt{h+4}} = \frac{h+4-9}{-(\sqrt{h+4}-3)} = \frac{\cancel{(\sqrt{h+4}-3)}(\sqrt{h+4}+3)}{-\cancel{(\sqrt{h+4}-3)}}$$

$$= \frac{\sqrt{h+4}+3}{-1} \xrightarrow[h\to 5]{} -6$$

Subst. gives $\frac{0}{0}$, indet.

9. $$\frac{\frac{1}{3+h} - \frac{1}{3}}{h} = \frac{\frac{3-(3+h)}{3(3+h)}}{h} = \frac{3-(3+h)}{h\cdot 3\cdot(3+h)}$$

$$= \frac{-\cancel{h}}{\cancel{h}\cdot 3(3+h)} = \frac{-1}{3(3+h)} \xrightarrow[h\to 0]{} -1/9$$

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10. $-1 \le \cos\frac{1}{x} \le 1$ $\quad x^2 \to 0$ $\quad -x^2 \le x^2 \cos\frac{1}{x} \le x^2$ Then we add 5.

and both go to 0 as $x \to 0$

11. $\lim_{x \to 4} \frac{|x-4|}{x^2-4x}$ Subst. gives $\frac{0}{0}$, indet.

$$\frac{|x-4|}{x^2-4x} = \begin{cases} -\frac{(x-4)}{x^2-4x} & x < 4 \\ \frac{x-4}{x^2-4x} & x \geq 4 \end{cases} = \begin{cases} -\frac{1}{x} & x < 4 \\ \frac{1}{x} & x > 4 \end{cases}$$

$$-\frac{(x-4)}{x^2-4x} = \frac{-1}{x} \xrightarrow[x \to 4^-]{} -1/4$$

$$\lim_{x \to 4^-} \frac{|x-4|}{x^2-4x} = -1/4 \qquad \lim_{x \to 4^+} \left(\frac{|x-4|}{x^2-4x}\right) = \frac{1}{4}$$

Since $-1/4 \neq \frac{1}{4}$ $\lim_{x \to 4} \frac{|x-4|}{x^2-4x}$ DNE