Solutions 151 WIR 6.1-6.2

1. $\overrightarrow{PQ} = 6\vec{i} + 2\vec{j}$ $P(3,-4)$ $Q(x,y)$

$x-3=6$ $y+4=2$

$x=9$ $y=-2$ $Q(9,-2)$

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2. $2\vec{a}+\vec{b} = -6\vec{i} - 8\vec{j} + 5\vec{i} - 2\vec{j}$

$= -\vec{i} - 10\vec{j}$

$|2\vec{a}+\vec{b}| = \sqrt{1+100} = \sqrt{101}$

$\vec{u} = \frac{-1}{\sqrt{101}}\vec{i} - \frac{10}{\sqrt{101}}\vec{j}$

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3. $-m-3n=5$ and $2m-4n=6$

2 times the 1st equation gives

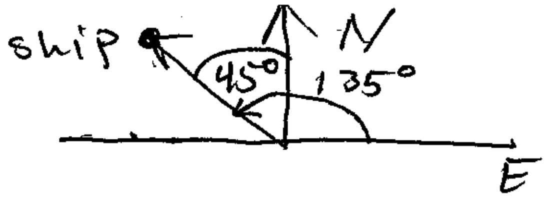
$-2m-6n=10$.

Add to the 2nd equation:

$$\begin{array}{r} -2m-6n=10 \\ 2m-4n=6 \\ \hline 0\ -10n=16 \end{array}$$ add

$\boxed{n=-1.6}$ Substitute this into the 1st equation to get $\boxed{m=-.2}$

4.



$$\vec{V}_{ship} = 20\cos 135^\circ\,\vec{i} + 20\sin 135^\circ\,\vec{j}$$

Let $\vec{V}_{man/ship}$ be the velocity of the man relative to the ship.

$$\vec{V}_{man/ship} = -3\vec{i}$$

$$\vec{V}_{man/water} = -3\vec{i} + 20\cos(135^\circ)\,\vec{i} + 20\sin 135^\circ\,\vec{j}$$

$$= (-3 - 10\sqrt{2})\,\vec{i} + 10\sqrt{2}\,\vec{j}$$

speed of man/water $= \sqrt{(-3-10\sqrt{2})^2 + (10\sqrt{2})^2}$

$\approx \boxed{22.25 \text{ mph.}}$

The polar angle for $\vec{V}_{man/water}$ is

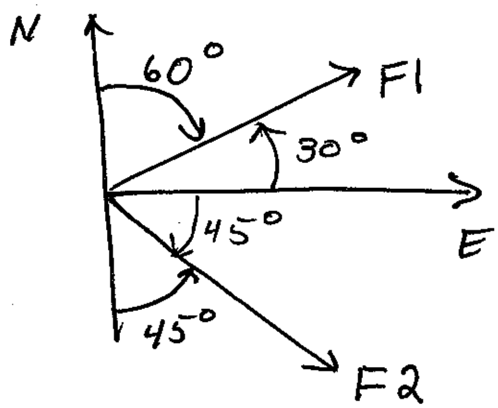
$\arctan\left(\frac{10\sqrt{2}}{-3-10\sqrt{2}}\right) + 180^\circ$ (Add 180° because the vector points into the 2nd quad. not the 4th)

$= 140^\circ$

The angle due north is $140^\circ - 90^\circ = \boxed{50^\circ}$

22.25 mph at 50° west of north.

5.



$$\vec{F} = \vec{F1} + \vec{F2}$$

$$\vec{F1} = 20\cos 30^\circ\,\vec{i} + 20\sin 30^\circ\,\vec{j}$$

$$\vec{F2} = 30\cos(-45^\circ)\,\vec{i} + 30\sin(-45^\circ)\,\vec{j}$$

$$\vec{F1} = 20\,\frac{\sqrt{3}}{2}\,\vec{i} + 20\cdot\frac{1}{2}\,\vec{j} = 10\sqrt{3}\,\vec{i} + 10\,\vec{j}$$

$$\vec{F2} = 30\cdot\frac{\sqrt{2}}{2}\,\vec{i} + 30\left(-\frac{\sqrt{2}}{2}\right)\vec{j} = 15\sqrt{2}\,\vec{i} - 15\sqrt{2}\,\vec{j}$$

$$\vec{F} = (10\sqrt{3} + 15\sqrt{2})\,\vec{i} + (10 - 15\sqrt{2})\,\vec{j}$$

$$|\vec{F}| = \sqrt{(10\sqrt{3} + 15\sqrt{2})^2 + (10 - 15\sqrt{2})^2} \approx 40 \text{ Newtons}$$

$$\measuredangle \vec{F} = \arctan\left(\frac{10 - 15\sqrt{2}}{10\sqrt{3} + 15\sqrt{2}}\right) \approx -16.2246^\circ$$

$$\text{acceleration} = \frac{\text{force}}{\text{mass}} \approx \frac{40}{50} = .8 \text{ m/sec}^2$$

at -16.2246° east of south

6. $\vec{u}\cdot(\vec{u}+\vec{v}) = \vec{u}\cdot\vec{u} + \vec{u}\cdot\vec{v}$

$$= |\vec{u}|^2 + 0 \quad = 1$$

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7. Find $\arccos\left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right)$

$|\vec{a}| = \sqrt{9+25} = \sqrt{34} \qquad |\vec{b}| = \sqrt{1+4} = \sqrt{5}$

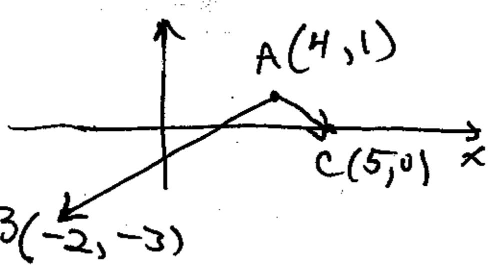
$\vec{a}\cdot\vec{b} = 3(-1) + (-5)(-2) = 7$

$\measuredangle \vec{a},\vec{b} = \arccos\left(\dfrac{7}{\sqrt{34}\sqrt{5}}\right) = \arccos\left(\dfrac{7}{\sqrt{170}}\right)$

$\approx 57.53^\circ$

or 1.004 radians

8.



$\measuredangle BAC$ is the angle between the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

$\overrightarrow{AB} = \langle -2-4, -3-1\rangle = \langle -6, -4\rangle$

$\overrightarrow{AC} = \langle 5-4, 0-1\rangle = \langle 1, -1\rangle$

$$\frac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{|\overrightarrow{AB}||\overrightarrow{AC}|} = \frac{-6+4}{\sqrt{36+16}\sqrt{1+1}} = \frac{-2}{\sqrt{104}} = \cos(\measuredangle BAC)$$

9, scalar projection $\vec{a} = 3\vec{i} + 4\vec{j}$ $\vec{b} = -2\vec{i} + \vec{j}$

$$= \text{comp}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \frac{-6+4}{\sqrt{9+16}} = \frac{-2}{5}$$

vector projetion

$$\text{proj}_{\vec{a}} \vec{b} = -\frac{2}{5} \frac{\vec{a}}{|\vec{a}|} = -\frac{2}{5}\left(\frac{3\vec{i} + 4\vec{j}}{5}\right)$$

$$= \frac{-6\vec{i} - 8\vec{j}}{25}$$

or $-\frac{6}{25}\vec{i} - \frac{8}{25}\vec{j}$

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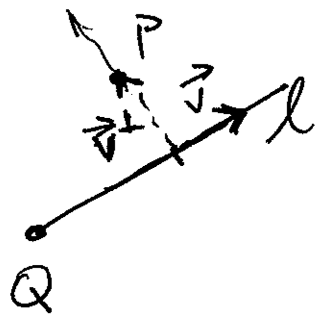
10. Work $= \vec{F} \cdot \vec{d} = 10(5)\cos 30^\circ$

$= 50 \frac{\sqrt{3}}{2}$ Joules

$= 25\sqrt{3}$ Joules

11.



Find Q on the line ℓ. (any Q will do)
Find a vector $\vec{v}^\perp$ which is perpendicular to ℓ.
Find $\left| \text{comp}_{\vec{v}^\perp} \overrightarrow{PQ} \right|$

To find $\vec{v}^\perp$, $\vec{v}^\perp = \langle 2, -1 \rangle$
from the coefficients in the line equation.
you must have it in the form $Ax + By = C$,
then $\vec{v}^\perp = \langle A, B \rangle$.

$Q(2,0)$ is on $2x - y = 4$

$\overrightarrow{PQ} = \langle 2-2, 0-5 \rangle = \langle 0, -5 \rangle$

$$\left| \text{comp}_{\vec{v}} \overrightarrow{PQ} \right| = \left| \frac{2(0) + (-1)(-5)}{\sqrt{4+1}} \right| = \frac{5}{\sqrt{5}}$$
$$= \sqrt{5}$$

Another way to find $\vec{v}^\perp$ is find another pt R on ℓ, find $\overrightarrow{RQ}^\perp$.

12. a) Find a vector $\perp$ to the line $\ell: 2x+3y=5$.

b) Show that $\langle 2,3\rangle \perp \ell$

c) Show that $\langle a,b\rangle \perp ax+by=c$

d) Find a vector // to $ax+by=c$

13. $y=3x+4$ a) Find a vector $\perp$ to the line.

b) // to the line

14. $\ell: \vec{\ell}(t)=(2t+1)\vec{i}+(3t-2)\vec{j}$

Find $\vec{v}$ a) // to $\vec{\ell}$ b) $\perp$ to $\vec{\ell}$.

c) Find the slope of line ℓ.