323 HONORS SPRING 2014, HOMEWORK DUE 2/11: PLEASE HAND IN SEPARATE FROM LEON HW

These problems are in addition to the assigned problems from Leon.

(1) A *convex linear combination* of vectors $\mathbf{v}_1, \cdots, \mathbf{v}_m$ is a vector of the form $\mathbf{w} = c_1\mathbf{v}_1 + \cdots + c_m\mathbf{v}_m$ where $c_j \geq 0$ and $\sum_j c_j = 1$. Prove that any convex linear combination of m vectors $\mathbf{v}_1, \cdots, \mathbf{v}_m$ in $\mathbb{R}^n$ is also a convex linear combination of no more than $n+1$ of the vectors $\mathbf{v}_1, \cdots, \mathbf{v}_m$.

(2) A function $f : \mathbb{R} \to \mathbb{R}$ is *periodic* with period p if $f(x+p) = f(x)$ for all $x \in \mathbb{R}$. Show that the space of periodic functions with period 2π is a linear subspace of the space of all functions $\mathbb{R} \to \mathbb{R}$.

(3) Let $C^0(\mathbb{R})_{2\pi}$ denote the vector space of continuous functions of period 2π. Show that the functions $\sin(x), \sin(2x), \sin(3x)$ are linearly independent elements of $C^0(\mathbb{R})_{2\pi}$. What about $\sin(x), \sin(2x), \cdots, \sin(kx)$ for any k?

(4) Show that the functions $1, \cos(x), \cos^2(x), \cos^3(x)$ are linearly independent vectors in $C^0(\mathbb{R})_{2\pi}$. Show that the function $\sin(\frac{x}{2})$ lies in their span.

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