323 HONORS SPRING 2014, HOMEWORK DUE 2/4

These problems are in addition to the assigned problems from Leon.

(1) Prove that the determinant of an $n \times n$ skew-symmetric matrix is zero when n is odd.

(2) Let x^i, $1 \le i \le n$ be variables, and let A be the $n \times n$ matrix with $A^i_{i+1} = x_i$, for $i < n$, $A^n_1 = x_n$, and all other entries zero. Show that $\det(I_n + A + A^2 + \cdots + A^{n-1}) = (1 - x_1 x_2 \cdots x_n)^{n-1}$. Hint: recall the identity $(1 + t + t^2 + \cdots + t^{n-2})(1 - t) = 1 - t^n$.

(3) Let x^i, y_j, $1 \le i, j \le n$ be variables, and let A be the $n \times n$ matrix with $A^i_j = \frac{1}{1 - x^i y_j}$. Compute $\det A$. Hint: use the Cauchy determinant for inspiration.

(4) Let A be an $n \times n$ matrix and λ a variable. Prove that $\det(A + \lambda I_n) = \lambda^n + \sum_{k=1}^n s_k \lambda^{n-k}$ where s_k is the sum of all $\binom{n}{k}$ principal minors of size k of A.

(5) Prove that the inverse of an upper triangular matrix is also upper triangular.

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