411 Honors homework due 2/8

A round-robin tournament of n contestants is a tournament in which each of the $\binom{n}{2}$ pairs of contestants play each other exactly once, with the outcome of any play being that one of the contestants wins and the other loses. For a fixed integer k, with $k < n$, determine whether it is possible that the outcome is such that, for every set of k players, there is a player who beat each member of that set. Show that if

$$\binom{n}{k}\left[1-(\frac{1}{2})^k\right]^{n-k} < 1$$

then such an outcome is possible.

Hint: Suppose that the results of the games are independent and that each game is equally likely to be won by either contestant.