Final, Math 411, sec 501 - Spring 2016

Printed name: ______________________________

The Aggie code of honor:"An Aggie does not lie, cheat or steal or tolerate those who do".

Signed name: ______________________________

Student ID number: ______________________________

Instructions:

- Read problems very carefully. If you have any questions raise your hand.
- The correct final answer alone is not sufficient for full credit - try to explain your answers as much as you can. This way you are minimizing the chance of losing points for not explaining details, and maximizing the chance of getting partial credit if you fail to solve the problem completely.
- You are allowed to use only the items necessary for writing. Notes, books, sheets, calculators, phones are **not** allowed, please put them away. If you need more paper please raise your hand (you can write on the back pages).
- There is no need to simplify numerical quantities like $4 \cdot 0.3 \cdot \frac{0.3+\frac{3}{4}}{\frac{7}{5}+\frac{23}{547}}$ or to reduce fractions like $\frac{3}{6}$ to lowest terms. There is also no need to simplify factorials such as 5! or binomial coefficients such as $\binom{7}{3}$.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total:	60	

1. A box contains $2n$ red and $2n$ blue toys. We select uniformly at random $2n$ toys from the box.
   (a) (5 points) Compute the probability that we selected equally many red and blue toys.
   (b) (5 points) Given that we selected more red than blue toys, compute the conditional probability that no blue toys were selected.

2. (10 points) Let X be a uniform random variable in the interval $[a, b]$ and Y an exponential random variable with parameter λ. If X and Y are independent, compute the PDF of $Z = Y - X$.

3. There are 4500 tiles delivered to a factory. Each tile is square shaped and the side length (in feet) is uniform in the interval $[0, 1]$, and is independent of side lengths of all other tiles. Let A be the total area of all tiles (that is sum of the areas of all tiles).

   (a) (5 points) Compute the expectation and the variance of A.

   (b) (5 points) Use the Central limit theorem to estimate the probability $\mathbf{P}(A \geq 1450)$.

4. (a) (5 points) Let $X_1, X_2, X_3, \ldots$ be an iid random variables, uniform in the interval $[1, 2]$. If

$$Y_n = (X_1 + X_2 + \cdots + X_n)^{1/n},$$

show that Y_n converges in probability to 1.

(b) (5 points) Let $U_1, U_2, U_3, \ldots$ be a sequence of iid random variables with mean 3 and variance 2. If

$$Z_n = \frac{U_1^2 + U_2^2 + \cdots + U_n^2}{U_1 + U_2 + \cdots + U_n},$$

show that Z_n converges with probability 1. What is the limit?

5. (a) (5 points) The transform of a random variable X is given by

$$M_X(s) = \frac{(e^s+1)^2(e^s+3)}{2^4}.$$

Compute $\mathbf{P}(X > 2)$.

(b) (5 points) The transform of a random variable X is given by

$$M_X(s) = \frac{2}{2-s}.$$

What is the variance of X?

6. (10 points) A machine prints out the number 1, 2, or 3 with equal probabilities. A machine starts printing numbers independently one after another and stops the first time it prints 1. Let X denote the number of two's it printed. Compute $\mathbf{E}[X]$ and $\mathrm{var}(X)$.