Final Exam, Math 170A - Fall 2014

Printed name: ____________________

Signed name: ____________________

Student ID number: ____________________

Instructions:

- Read problems very carefully. If you have any questions raise your hand.
- The correct final answer alone is not sufficient for full credit - try to explain your answers as much as you can (except for Problem 1). This way you are minimizing the chance of losing points for not explaining details, and maximizing the chance of getting partial credit if you fail to solve the problem completely. However, try to save enough time to seriously attempt to solve all the problems.
- You final answers need to be simplified only if this is required in the statement of the problem. Otherwise, you can leave them in any form you wish.
- You are allowed to use only the items necessary for writing. Notes, books, sheets, calculators, phones are not allowed, please put them away. If you need more paper please raise your hand (you can write on the back pages).

Question	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
7	5	
Total:	35	

1. (5 points) For every claim write if it is true or false. Write a short explanation.

   (a) The sum of two continuous random variables is a continuous random variable.

   (b) Let X, Y be random variables. Then $\mathbf{Var}[X+Y] = \mathbf{Var}[X] + \mathbf{Var}[Y]$.

   (c) Let X, Y, Z be random variables, then $\mathbf{E}[X+Y|Z] = \mathbf{E}[X|Z] + \mathbf{E}[Y|Z]$.

   (d) Let $X_i \sim exp(1)$ be independent and $A_n = \{X_n > \alpha \log(n)\}$, then $\mathbf{P}[\limsup A_n] = 1$ if and only if $\alpha \geq 2$.

   (e) Let $X, Y \sim \mathcal{N}(\mu, \sigma^2)$ be independent random variables, then $\mathbf{P}[X > Y]$ doesn't depend on μ and σ^2.

2. (5 points) Suppose $a < b$. Let X be a continuous uniform distribution on the interval $[a, b]$, and given $X = x$, let Y be a continuous uniform distribution on the interval $[a, x]$. Find the PDF of Y and $E(Y)$.

3. (5 points) A person places randomly n letters in to n envelops . What is the probability that no letter reached its destination.

4. (5 points) Each of k jars contains m white balls and n black balls. A ball is randomly chosen from jar 1 and placed in jar 2, then a ball is randomly chosen from jar 2 and placed in jar 3, etc. Finally a ball is randomly chosen from jar k. Show that the probability that the last ball is white is the same as the probability that the first ball is white.
Hint: Use induction.

5. (5 points) A food store just got a delivery of 3000 potatoes. It is known that each potato is rotten with probability 0.2. What is the expected number of healthy potatoes? Use the normal approximation to estimate the probability that there are at least 2950 healthy potatoes. (Write solution in terms of Φ).

6. (5 points) Let the random variables X and Y be jointly continuous with the joint PDF

$$f_{X,Y}(x,y) = \begin{cases} ke^{-(ax+by)}, & \text{if } x > 0, y > 0, \\ 0, & \text{otherwise,} \end{cases}$$

where $a, b > 0$ and k is a constant.

1. Find k.
2. Are X and Y independent?

7. (5 points) Let $\{X_i\}_{i=1}^{\infty}$ be independent $exp(1)$ random variables. Denote $Y_n = \min\{X_1, X_2, \ldots, X_n\}$. What is the probability that $Y_n > 1$ for infinitely many n's .