(1) Let $\phi = dx^1 \wedge dx^{m+1} + \cdots + dx^m \wedge dx^{2m} \in \Lambda^2 \mathbb{R}^{2m*}$. Explicitly compute the Lie algebra of endomorphisms annhilating ϕ, $\mathfrak{sp}(\mathbb{R}^{2m}, \phi) \subset \mathfrak{gl}_{2m}$.

(2) Let (M, ϕ) be a $2m$-dimensional manifold with $\phi \in \Omega^2(M)$ non-degenerate, i.e. $\phi^{\wedge m} \in \Omega^{2m}(M)$ is non-vanishing. (Such is called an *almost symplectic manifold*. Show that the analog of the fundamental Lemma of Riemannian geometry fails in the sense that on $\mathcal{F}_{SP(V)}(M)$ (the fundle of frames $(\eta^1, \cdots, \eta^{2m})$ such that $\phi = \eta^1 \wedge \eta^m + \cdots + \eta^m \wedge \eta^{2m}$), there does not in general exist a $\mathfrak{sp}(V)$-valued 1-form θ such that $d\eta = -\theta \wedge \eta$ unless $d\phi = 0$. When $d\phi = 0$, (M, ϕ) is called a *symplectic manifold*.

(3) Let (M, ϕ) be a symplectic manifold. Compute $d\theta + \theta \wedge \theta$. Use this to prove Darboux's theorem (CFB Thm. 1.9.14).