666 HOMEWORK DUE 10/15/7

(1) Let X be a compact complex manifold and let $Y \subset X$ be a complex submanifold. For each open set $U \subseteq X$, let $\mathcal{I}_Y(U)$ be the ideal in the ring $\mathcal{O}_X(U)$ consisting of those functions which vanish at all points of $Y \cap U$. Show that the presheaf $U \mapsto \mathcal{I}_Y(U)$ is a sheaf, in fact a subsheaf of $\mathcal{O}_X$.

(2) Show that the quotient sheaf $\mathcal{O}_X/\mathcal{I}_Y$ is isomorphic to $i_*(\mathcal{O}_Y)$, where $i : Y \to X$ is the inclusion map.

(3) Now let $X = \mathbb{CP}^1$ and let Y be the union of two distinct points, $p, q \in \mathbb{P}^1$. Then, letting $\mathcal{F} = (i_*\mathcal{O}_p \oplus i_*\mathcal{O}_q)$, there is an exact sequence of sheaves on X

$$0 \to \mathcal{I}_Y \to \mathcal{O}_X \to \mathcal{F} \to 0.$$

Show that the induced map $\Gamma(X, \mathcal{O}_X) \to \Gamma(X, \mathcal{F})$ is not surjective. (These first three problems taken from Hartshorne, p69.)

(4) Calculate the Cech cohomology groups of the following sheaves on $\mathbb{P}^1$ (or $\mathbb{P}^n$ if you are more ambitious): the constant sheaf $\mathbb{C}$, the sheaf holomorphic sections of $K_{\mathbb{P}^n} = \Lambda^n T^*\mathbb{P}^n$.

E-mail address: jml@math.tamu.edu