

#9 $-4x^2 - 12x - 106$

Math 150 Sections 513-518 Exam 2b - Jean Marie Linhart Fall 2009

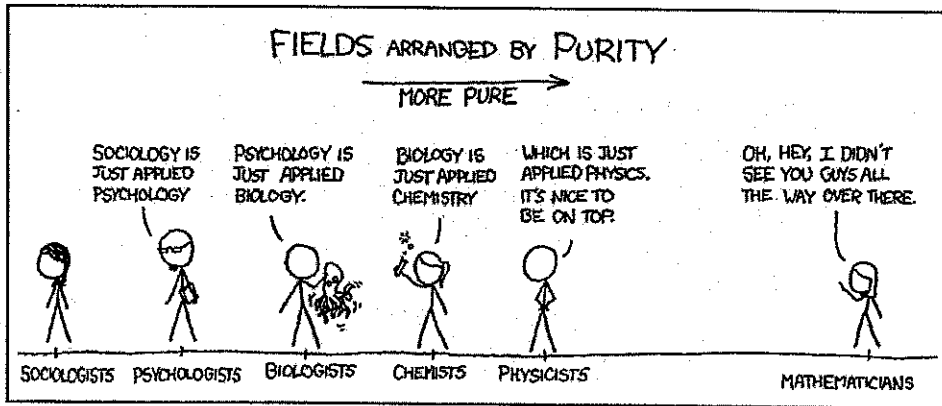
"An Aggie does not lie, cheat, or steal or tolerate those who do"
On my honor as an Aggie, I have neither given nor received unauthorized aid
on this exam.

Printed name: Rubric

Signature: _____

Circle your section:

513 514 515 516 517 518



xkcd.com

- Read each question carefully.
- Show your work!
- On the multiple choice questions, circle the correct multiple choice answer.
- You may not use any notes, a calculator, or your book.
- Your cellphone must be turned off and put away during this exam.
- You may not collaborate with your neighbors on this exam.
- You must show all appropriate work to receive credit, especially partial credit.
- If you use a formula, WRITE IT DOWN.
- The instructor will provide additional scratch paper if needed.
- You must put your name on any scratch paper and hand it in with your exam.
- The only things with you at your desk are pencils, erasers, student ID, water bottles. No hats, no sunglasses, no wallets, no cellphones.
- GOOD LUCK!!!!!!!

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Math Facts You Might Find Useful:

$$13^2 = 169 \quad 14^2 = 196 \quad 15^2 = 225$$

$$16^2 = 256 \quad 17^2 = 289 \quad 18^2 = 324$$

$$19^2 = 361 \quad 20^2 = 400 \quad 21^2 = 441$$

$$22^2 = 484 \quad 23^2 = 529 \quad 24^2 = 576$$

$$25^2 = 625$$

$$2^3 = 8 \quad 2^4 = 16 \quad 2^5 = 32$$

$$2^6 = 64 \quad 2^7 = 128 \quad 2^8 = 256$$

$$2^9 = 512 \quad 2^{10} = 1024$$

Prime numbers less than 20: 2 3 5 7 11 13 17 19

A number is divisible by 3 if its digits add up to a number divisible by 3.

A number is divisible by 9 if its digits add up to a number divisible by 9.

This is not true for other factorizations!

Difference quotient:

$$\frac{f(x+h) - f(x)}{(x+h) - x}$$

First 3 letters of last name:

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Printed name: _____

Circle your section:

513 514 515 516 517 518

1. Let $f(x) = x^2 + 4$, $g(x) = -x - 2$, $h(x) = \sqrt{x-2}$, and $k(x) = \frac{1}{x-4}$

a. (2 points) Find $\left(\frac{g}{f}\right)(x)$ and its domain in interval notation.

$$\frac{g}{f}(x) = \frac{-x-2}{x^2+4} + 1$$

domain $(-\infty, \infty) + 1$

b. (4 points) Find $(k \circ h)(x)$ and its domain in interval notation.

$$k \circ h(x) = \frac{1}{\sqrt{x-2} - 4} + 1$$

Domain $x \geq 2 + 1$

$$\sqrt{x-2} - 4 \neq 0 + 1$$

$$\sqrt{x-2} \neq 4$$

$$x-2 \neq 16$$

$$x \neq 18$$

Domain $[2, 18) \cup (18, \infty) + 1$

c. (3 points) Find $(h \circ f \circ g)(2)$

$$g(2) = -4 + 1$$

$$f(-4) = 20 + 1$$

$$h(20) = \sqrt{18} = \boxed{3\sqrt{2}} + 1$$

d. (3 points) Find $(f+h)(x)$ and its domain in interval notation.

$$(f+h)(x) = x^2 + 4 + \sqrt{x-2} + 1$$

Domain $[2, \infty) + 2$

2. (5 points) Let $(m \circ h)(x) = (x^2 + 5)^3 + 3(x^2 + 5)^2$. Identify $m(x)$ and $h(x)$.

a. $m(x) = x^3 + 3x^2$, $h(x) = x^2 + 5$

b. $m(x) = x^2 + 5$, $h(x) = x^3 + 3x^2$

c. $m(x) = x^3 + 3x^2$, $h(x) = x + 5$

d. $m(x) = x^3 + 3x$, $h(x) = x^2 + 5$

e. None of these

$$\begin{array}{r} 44 \\ 3 \\ \hline 132 \\ -16 \\ \hline 2 \overline{) 116} \\ 2 \overline{) 58} \\ 29 \end{array}$$

3. (4 points) What are all the zeros (real or complex) of

$$f(x) = 11x^2 + 4x + 3$$

$$\frac{-4 \pm \sqrt{16 - 132}}{22} = \frac{-4 \pm \sqrt{-116}}{22} =$$

$$\frac{-4 \pm 2i\sqrt{29}}{22}$$

$$\boxed{\frac{-2 \pm i\sqrt{29}}{11}} \quad +2$$

(2 points) Are these zeros real or complex?

Complex +2

4. (4 points) Find the value of a so that the line $6x - 10y = 2$ is parallel to the line $ax - 4y = -9$.

Or:

$$-10 \cdot \frac{4}{10} = -4 \times 2$$

$$6 \cdot \frac{4}{5} = \frac{12}{5} = a \times 2$$

$$\begin{aligned} -10y &= -6x + 2 \\ y &= \frac{3}{5}x - \frac{1}{5} \end{aligned}$$

$$\begin{aligned} -4y &= -ax - 9 \\ y &= \frac{a}{4}x - 9 \end{aligned}$$

$$\frac{a}{4} = \frac{3}{5}$$

$$\boxed{a = \frac{12}{5}}$$

5. (5 points) Find the domain of

$$f(x) = \frac{\sqrt{x^2 - 3x - 28}}{x + 9}$$

a. $(-\infty, -9] \cup [-9, -4] \cup [7, \infty)$

b. $(-\infty, -9] \cup [-4, 7]$

c. $(-\infty, -9) \cup (-9, -4] \cup [7, \infty)$

d. $(-9, -4] \cup [7, \infty)$

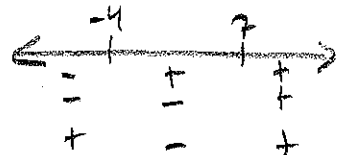
e. None of these

$$x \neq -9$$

$$\sqrt{(x-7)(x+4)}$$

$$(x-7)(x+4) \geq 0$$

$$(-\infty, -9) \cup (-9, -4] \cup [7, \infty)$$



6. (10 points) Evaluate the difference quotient of the following function then simplify.

$$f(x) = \frac{-5}{3x^2 - 7}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{-5}{3(x+h)^2 - 7} - \frac{-5}{3x^2 - 7}}{h}$$

$$= \frac{\frac{-5}{3(x^2 + 2hx + h^2) - 7} + \frac{5}{3x^2 - 7}}{h}$$

+2
or
multiply
through
to eliminate
denoms.

$$= \frac{\frac{-5(3x^2 - 7)}{3(x^2 + 2hx + h^2) - 7} + \frac{5(3x^2 + 6xh + 3h^2 - 7)}{3x^2 - 7}}{h}$$

$$= \frac{-15x^2 + 35 + 15x^2 + 30xh + 15h^2 - 35}{[3(x+h)^2 - 7][3x^2 - 7]} + 1$$

$$= \frac{30xh + 15h^2}{[3(x+h)^2 - 7][3x^2 - 7]} \cdot \frac{1}{h} = \frac{30x + 15h}{[3(x+h)^2 - 7][3x^2 - 7]}$$

7. (4 points) Describe the end behavior of the polynomial

$$p(x) = -9x^{86} - 54x^{93} - 68x$$

as $x \rightarrow -\infty$, $p(x) \rightarrow -\infty$ +2

as $x \rightarrow \infty$, $p(x) \rightarrow \infty$ +2

8. (5 points) Which points are on the graph of $xy^2 = x^4 + x$?

~~A = (1, 1)~~ B = (0, 0) ✓ C = (1, $\sqrt{2}$) ✓

a. only A and B

b. only A and C

c. only B and C

d. A, B and C

e. only B

$1(2) = 1 + 1$ ✓

$$-4(x^2 + \frac{3}{2}x + \frac{9}{4}) - 106 + \frac{9}{4}$$

$$-4(x + \frac{3}{4})^2 - \frac{424}{4} + \frac{9}{4} = -4(x + \frac{3}{4})^2 - \frac{415}{4}$$

9. Given the following function:

$$f(x) = -4x^2 - 6x - 106$$

a. (4 points) Complete the square to find the vertex

$$-4(x^2 + \frac{3}{2}x + \frac{9}{4}) - 106 + \frac{9}{4}$$

$\frac{3}{4} = p$

$$-4(x + \frac{3}{2})^2 - 97$$

vertex $(-\frac{3}{2}, -97)$

b. (2 points) What the value of the function at its minimum? If there is no minimum value, write "NONE".

NONE +2

c. (2 points) What is the value of the function at its maximum? If there is no maximum value, write "NONE".

-97 +2 (ans from a)

d. (2 points) Write the equation for its axis of symmetry.

$x = -\frac{3}{2}$ +2 (ans from a)

10. (10 points) It takes Timo 5 minutes longer than Jean Marie to make a dinner salad. Together it takes them 6 minutes. How long does it take Timo to do the job by himself (do not forget the units)? Define the variables before solving the problem.

Time for Timo = t +2
 Time for JM = $t - 5$

$$\frac{1}{t} + \frac{1}{t-5} = \frac{1}{6} \quad +2$$

$$6[t-5+t] = t^2 - 5t \quad +1$$

~~$$6t^2 - 30t - 2t + 5 = 0$$

$$6t^2 - 32t + 5 = 0$$~~

$$12t - 30 = t^2 - 5t$$

$$t^2 - 17t + 30 = 0$$

$$(t-15)(t-2) = 0$$

$$t = 15 \quad t = 2$$

15 minutes

+2 (1 for 15 minutes)

32 ± ✓

11. (7 points) Find f^{-1} for $f(x) = (x+5)^3 - 6$. What are the domain and range of f and f^{-1} ?

Domain $f = (-\infty, \infty)$ Range $f = (-\infty, \infty)$
 Domain $f^{-1} = (-\infty, \infty)$ Range $f^{-1} = (-\infty, \infty)$

$$x = (y+5)^3 - 6 \quad +1$$

$$x+6 = (y+5)^3 \quad +1$$

$$\sqrt[3]{x+6} = y+5 \quad +1$$

$$\sqrt[3]{x+6} - 5 = y \quad +1$$

$$f^{-1}(x) = \sqrt[3]{x+6} - 5 \quad +1$$

12. (4 points) Suppose we are given the graph of a function $f(x)$. How must we transform the graph of f to look like the graph of g if $g(x) = -4f(x+2)+3$? List the specific graph transformations that are required to turn $f(x)$ into $g(x)$. Leave any extra lines blank.

Any order

Transformation 1: Reflect f about x axis

Transformation 2: Scale by 4

Transformation 3: Move 3 units up

Transformation 4: Move 2 units left

Transformation 5: _____

Transformation 6: _____

1 pt each

13. (5 points)

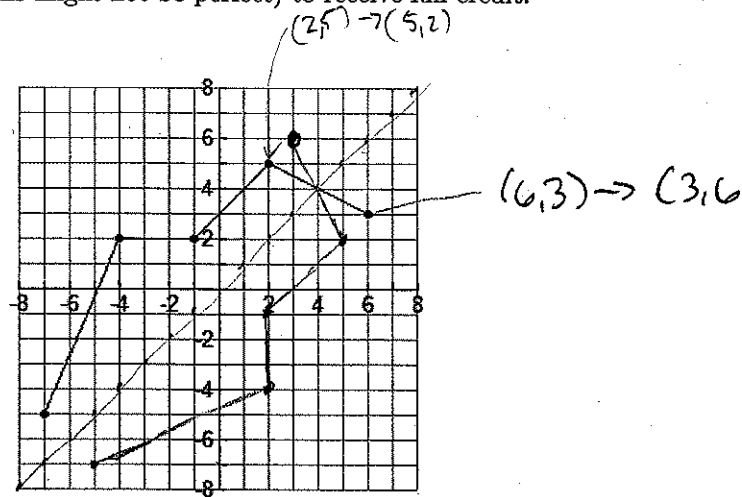
$$f(x) = \begin{cases} \frac{1}{(x+3)^2} + 1 & \text{if } x < -1 \\ 3 & \text{if } -1 < x \leq 3 \\ \frac{1}{(x-1)^2} - 2 & \text{if } x > 3 \end{cases}$$

Find the values of $f(-5)$ and $f(3)$, respectively.

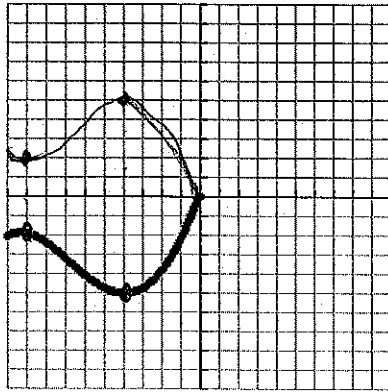
- a. $\frac{5}{4}, \frac{-7}{4}$ b. $\frac{-7}{4}, \frac{-5}{4}$ c. $\frac{5}{4}, 3$ d. $3, \frac{-7}{4}$ e. $\frac{7}{4}, \frac{5}{4}$

$\frac{1}{4} + 1 = \frac{5}{4}$

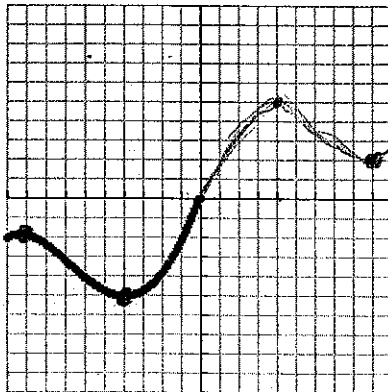
14. (4 points) Sketch the inverse relation for the given graph on the same coordinate plane. Your graph must be accurate (though your drawing skills might not be perfect) to receive full credit.



15. (3 points) Complete the graph so that it is symmetric about the x-axis.



16. (3 points) Complete the graph so that it is symmetric about the origin.



17. (8 points) Sketch any function that is/has

- increasing on the intervals $(-\infty, -5)$ and $(5, \infty)$ +2 pts
- undefined at $x = -5$. +1
- decreasing on $(-5, 5)$ +1
- x -intercepts at $-8, -2, 8$ +3 (each)
- a y -intercept of -2 +1

You are not being graded on your artistic skills. Just draw a function that works with all the information above.

