Math 150 Sections 513-518 Exam 3a – Jean Marie Linhart Fall 2009

"An Aggie does not lie, cheat, or steal or tolerate those who do"
On my honor as an Aggie, I have neither given nor received unauthorized aid on this exam.

Printed name: Key

Signature: _____

Circle your section:

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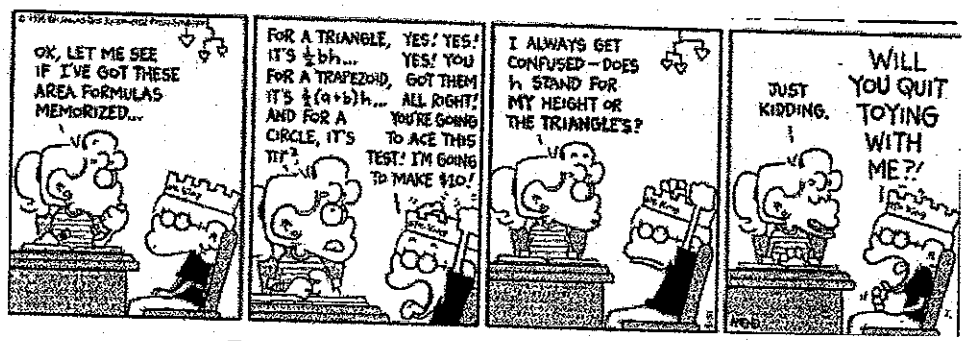


Fox Trot by Bill Amend http://www.gocomics.com/foxtrot

- Read each question carefully.
- Show your work!
- On the multiple choice questions, circle the correct multiple choice answer.
- You may not use any notes or your book.
- Your cellphone must be turned off and put away during this exam.
- You may not collaborate with your neighbors on this exam.
- You must show all appropriate work to receive credit, especially partial credit.
- If you use a formula, WRITE IT DOWN.
- The instructor will provide additional scratch paper if needed.
- You must put your name on any scratch paper and hand it in with your exam.
- The only things with you at your desk are pencils, erasers, student ID, water bottles. **No hats, no sunglasses, no wallets, no cellphones.**
- GOOD LUCK!!!!!!!

First 3 letters of last name:

Printed name:_____

Circle your section:

513 514 515 516 517 518

1. (5 points) Find the horizontal and vertical asymptotes of

$$f(x) = \frac{3x^2 - 3x - 18}{5x^2 - 10x - 40}$$

$\frac{3(x^2 - x - 6)}{5(x^2 - 2x - 8)} = \frac{3(x-3)(x+2)}{5(x-4)(x+2)}$

A) horizontal asymptote at $y = 4$, vertical asymptote at $x = 3/5$

B) horizontal asymptote at $y = 3/5$, vertical asymptotes at $x = -2$ and $x = 4$

C) horizontal asymptotes at $y = -2$, $y = 4$, vertical asymptote at $x = 3/5$

D) horizontal asymptote at $y = 3/5$, vertical asymptote at $x = 4$

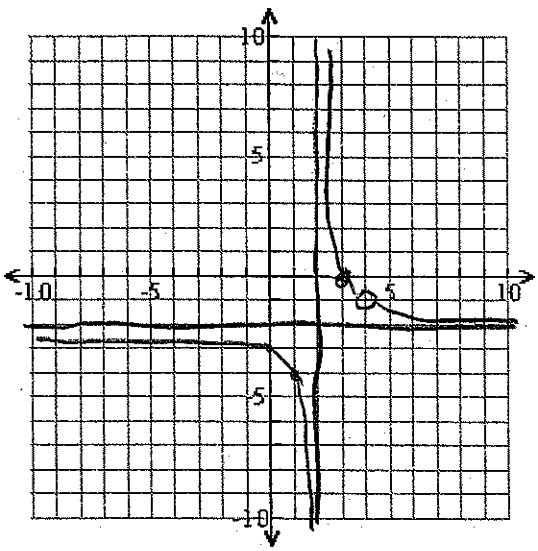
E) None of these

no asymptote at $x = -2$

2. (5 points) Graph

$$f(x) = \frac{4x - 16}{2x^2 - 12x + 16} - 2$$

below. Clearly identify the horizontal and vertical asymptotes and any holes.



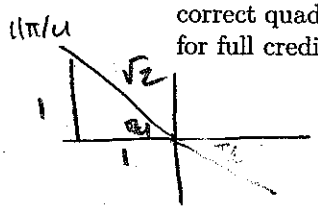
$\frac{4(x-4)}{2(x^2 - 6x + 8)} - 2$

$\frac{4(x-4)}{2(x-4)(x-2)} - 2$

@ $x = 4$

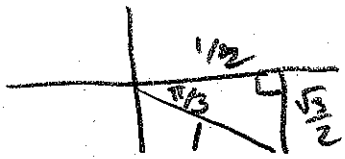
$\frac{2}{2} - 2 = -1$

3. (3 points) Find the exact value of $\tan\left(\frac{11\pi}{4}\right)$. Draw this angle in the correct quadrant and draw the reference triangle with lengths on all sides for full credit.



−1

4. (3 points) Find the exact value of $\cos\left(\frac{5\pi}{3}\right)$ Draw this angle in the correct quadrant and draw the reference triangle with lengths on all sides for full credit.



1/2

5. (10 points total) Given the half-life of a radioactive substance is 17 years; we want to find how long will it takes for 300 grams of this substance to decay to 200 grams.

A) (2 points) First write down the general form of the equation for radioactive decay.

$P(t) = Ae^{kt}$ or $P(t) = P(0)e^{kt}$

B) (1 point) Now write an equation incorporating the half-life information; one that you would use to calculate the decay constant from the half-life.

$e^{17k} = 1/2$

C) (3 points) Find the exact value of the radioactive decay constant.

$17k = \ln 1/2$

$k = \frac{\ln(1/2)}{17}$

D) (1 point) Use the decay constant from part C) and the information in the original question to write an equation that can be used to find the time requested.

$300\, e^{\frac{\ln(1/2)}{17}t} = 200$

E) (3 points) Solve the equation you wrote in part D) for the requested time. Either find an exact answer or round the time to 3 decimal places.

$e^{\frac{\ln(1/2)}{17}t} = 2/3$

$\frac{\ln(\frac{1}{2})}{17}t = \ln(2/3)$

$t = 17\frac{\ln(2/3)}{\ln(1/2)} \approx 9.944$

6. (5 points) The number of bacteria in a petri dish is given by $P(t) = 0.25 \cdot 2^{0.75t}$ **thousand** bacteria, where t is in days. Find the exact time when the population will reach 15,000 bacteria. The answer choices below are in days.

Hints: Note the units on the problem. It may be helpful to use fractions where possible.

A) $\frac{4\ln 60}{3\ln 2}$ B) $\frac{3\ln 60}{4\ln 2}$ C) $\frac{4\ln 2}{3\ln 60}$ D) $\frac{4\ln 60000}{3\ln 2}$ E) $\frac{3\ln 15}{4\ln 2}$

$\frac{1}{4} 2^{3/4 t} = 15$ (thousands- note units)

$2^{3/4 t} = 60$

$\frac{3}{4} t \ln 2 = \ln 60$

$t = \frac{4 \ln 60}{3 \ln 2}$

7. (5 points) Find the area of the sector subtended by a central angle of $\frac{9\pi}{10}$ radians in a circle with radius 30 cm.

A) 405π cm^2 B) 810π cm^2 C) 13.5π cm^2 D) 27π cm^2 E) none of these

$\frac{\pi(30^2) \cdot \frac{9\pi}{10}}{2\pi}$ cm^2 $= 405\pi$ cm^2

$\begin{array}{r} 45 \\ \times\ 9 \\ \hline 405 \end{array}$

8. (6 points; 3 each) Convert the following angle measures. Show your work by writing down the multiplication with the appropriate unit conversion factor. Give exact answers.

$\frac{13\pi}{30}$ radians to degrees $\frac{13\pi}{30} \cdot \frac{180^\circ}{\pi} =$

$\begin{array}{r} 13 \\ \times\ 6 \\ \hline 78 \end{array}$

$\frac{13\pi}{30}$ radians $=$ 78 degrees

35° to radians $\frac{7}{35^\circ} \cdot \frac{\pi}{180} =$ $\frac{7}{36}$

$35^\circ = \frac{7}{36}\pi$ radians

9. (5 points) Expand

$$f(x) = \log_4\left(\frac{(x+7)^3}{5x^2}\right)$$

A) $3\log_4(x+7) - \log_4 5 + 2\log_4 x$
B) $3\log_4 x + 3\log_4 7 - \log_4 5 + 2\log_4 x$
C) $3\log_4 x + 3\log_4 7 - \log_4 5 - 2\log_4 x$
D) $3\log_4(x+7) - \log_4 5 - 2\log_4 x$
E) None of these

$\log_4 (x+7)^3 + (\log_4 5x^2)$
$3\log_4(x+7) - [\log_4 5 + 2\log_4 x]$
$3\log_4(x+7) - \log_4 5 - 2\log_4 x$

10. (5 points) Consider the following three statements.

I. $e^{\ln x^{-3}} = x^{-3}$. T

II. The domain of $y = \log_4(13-x)$ is $(-\infty, 13]$. F should be $-(-\infty, 13)$

III. $\log_7(5) = \frac{\ln 5}{\ln 7}$ T

A. Only I and II are true.
B. Only I and III are true. ← 5 pts
C. Only II and III are true.
D. I, II, and III are true. — 3 pts
E. I, II, and III are false.

11. (8 points) Exactly solve the equation for x.

$$7e^{6-2x} + 2 = 16$$

$7e^{6-2x} = 14$ +1
$e^{6-2x} = 2$ +1
$6 - 2x = \ln 2$ +2
$-2x = \ln 2 - 6$ +1
$x = 3 - \frac{\ln 2}{2}$ +3

$-\frac{\ln 2}{2} + 3$

12. (5 points) In interval notation, what is the domain of the function?

$$g(x) = \frac{\sqrt{x-7}}{\log_{15}(16-x)}$$

$x \geq 7$ $\quad 16-x > 0$ $\quad 16 > x$

$\log_{15}(16-x) \neq 0$ $\quad 16-x \neq 15^0$ $\quad 16-x \neq 1$ $\quad 15 \neq x$

+1 +2 +2

Domain: $[7, 15) \cup (15, 16)$

13. (5 points) A baker wants to make brownies with 400g of a 62% whole wheat flour mixture. On hand, he has an 18% whole wheat flour mixture that he uses for cakes, and a 74% whole wheat flour mixture he uses for cookies.. He wants to mix these two prepared flour mixtures together to get the flour mixture specified for his brownies. Which system of equations can be used to solve this problem?

.62 (400)

$.18x + .74y = 248$

A) $\begin{aligned} x + y &= 62 \\ 62x + 18y &= 29600 \end{aligned}$

B) $\begin{aligned} x + y &= 400 \\ 18x + 74y &= 248 \end{aligned}$

C) $\begin{aligned} x + y &= 400 \\ 18x + 74y &= 24800 \end{aligned}$

D) $\begin{aligned} x + y &= 62 \\ 18x + 74y &= 400 \end{aligned}$

E) None of these

$x + y = 400$

$18x + 74y = 24800$

14. (6 points) What value of a makes the system of equations

$$\begin{aligned} 4x - 7y &= 21 \\ ax + 2y &= 13 \end{aligned}$$

have no solution?

$-7y = 21 - 4x$

$y = -3 + \frac{4}{7}x$

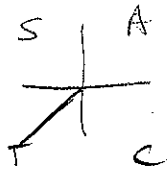
$2y = 13 - ax$

$y = \frac{13}{2} - \frac{a}{2}x$

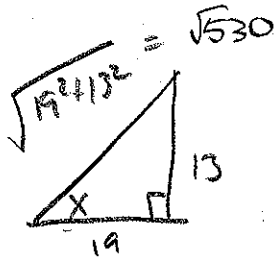
$-\frac{a}{2} = \frac{4}{7}$

$a = -\frac{8}{7}$

15. (10 points, 2 each) If $\cot x = \frac{19}{13}$ where $\sin x < 0$, find the exact values of all the trigonometric functions.



Quadrant 3



$\sin x = -\frac{13\sqrt{530}}{530}$

$\cos x = -\frac{19\sqrt{530}}{530}$

$\tan x = \frac{13}{19}$

$\csc x = -\frac{\sqrt{530}}{13}$

$\sec x = -\frac{\sqrt{530}}{19}$

16. (5 points)

$$f(x) = \left(\frac{1}{2}\right)^{x-3} + 1$$

Which of the following three statements are true?

I. $f(x)$ is increasing. F

II. $f(x)$ has an intercept at $(0, 9)$. T

III. $f(x)$ has a horizontal asymptote at $y = 1$. T

A) Only I is true. B) Only II is true.

C) Only III is true. D) Only I and III are true.

E) Only II and III are true.

17. (5 points) Find all solutions for x and y

$$\begin{aligned} 2(y^2 + 2y - 1 &= -\log_2 x) \\ -y^2 - 5y + 2 &= 2\log_2 x \end{aligned}$$

A) $(2, 0)$ and $\left(\frac{1}{4}, 1\right)$ B) $(0, 2)$ and $\left(1, \frac{1}{4}\right)$

C) Only $\left(1, \frac{1}{4}\right)$ D) Only $(0, 2)$ E) Only $(2, 0)$

$y^2 - y = 0$

$y(y-1) = 0$

$y = 0 \quad y = 1$

$y = 0$

$-1 = -\log_2 x$

$x = 2^1$

$y = 1$

$2 = -\log_2 x$

$x = 1/4$

18. (5 points) If

$$y = 4\sin(-x - \pi) + 1$$

what are the amplitude, period, reflection, and vertical shift?

A) Amplitude 4, period ~~X~~, reflected over y-axis, vertical shift 1 unit up.

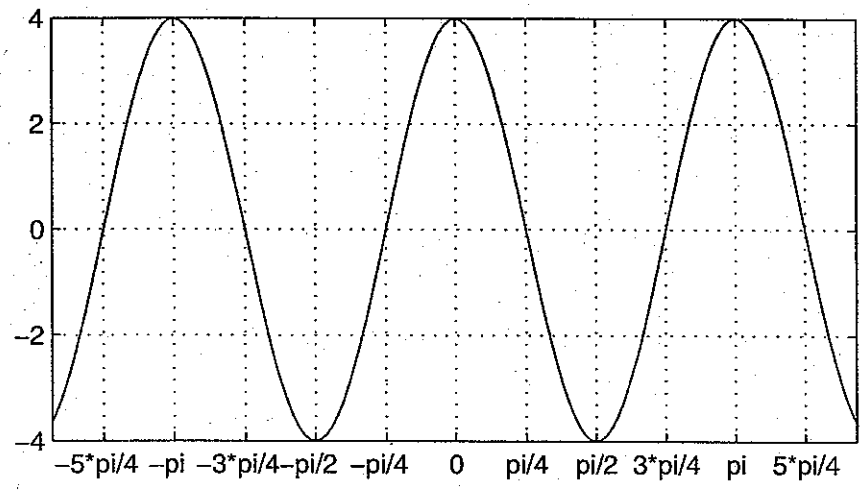
B) Amplitude 4, period 2π, reflected over the ~~x~~-axis, vertical shift 1 units up.

C) Amplitude ~~1~~, period 2π, reflected over the $y-$axis, vertical shift ~~1~~ units up.

D) Amplitude ~~1~~, period ~~π~~, reflected over the $y-$axis, vertical shift ~~1~~ units up.

E) None of these

19. (5 points) Write a function of the form $f(x) = a\sin[k(x-b)]$ whose graph is shown below where a, b and k are positive and b is as small as possible.



period π

$K = 2$

$f(x) = \dfrac{4\sin 2(x - \frac{3\pi}{4})}{}$

1 ↓ 2 ↓ 1