"An Aggie does not lie, cheat, or steal or tolerate those who do"

On my honor as an Aggie, I have neither given nor received unauthorized aid on this exam.

Printed name:__________________________

Signature:______________________________

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You may find the following useful:

$$\frac{d[\arctan(u)]}{du} = \frac{1}{1+u^2}$$

Recall $\tan[\arctan(u)] = u$.

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- You may use your 4x6 inch notecard for this exam. You must hand it in with your exam.
- You may not use any other notes, a calculator, or your book.
- You may not collaborate with your neighbors on this exam.
- You must show all appropriate work to receive credit, especially partial credit.
- If you use an integrating factor, identify it!
- The instructor will provide additional scratch paper if needed.
- Read each question carefully.

1) (20 points) Find an implicit solution to the following:

$$\frac{dy}{dx} = -\frac{y^3 + 4e^x y}{2e^x + 3y^2}$$

2) (15 points) Find an explicit solution to the following:

$$x\frac{dy}{dx} - y = x^3 \sin(2x) \qquad y(\pi) = 0$$

3) (2 points) Identify the dependent and independent variables in:

$$\frac{dy}{d\theta} + \frac{y}{\theta} = -6\theta y^{-2} \qquad y(1) = 0$$

Independent variable:________________ Dependent variable:________________

(2 points) What kind of a differential equation is this?

(15 points) Find an explicit solution to the above initial value problem.

4) (10 points) Use separation of variables to find the solution to

$$\frac{dy}{dx} = 15x^4(y-1)^{2/3} \qquad y(0) = 1$$

(5 points) Is the solution you found by separation of variables a unique solution? If so, give a theorem stating why. If not, give another solution.

5) (5 points) An ice cube (all sides are square) melts with the change in its volume proportional to its surface area. Assume it remains cubical as it melts. Initially it is 1 cubic inch in **volume**, and after 5 minutes it is 1/8 cubic inch in **volume**. Set up a differential equation representing the change in the length of a side of the ice cube as a function of time. Note: a cube has 6 faces that are all perfect squares of the same size.

(5 points) Solve the differential equation from above and use the information given to write an explicit equation for the **length of a side of the cube** as a function of time.

(5 points) Use your solution, above, to calculate when the ice cube completely disappears.

6) (12 points) Find any explicit solution for

$$\frac{dy}{dx} = 2xy^2 + 2x \qquad y(\sqrt{\pi}) = 0$$

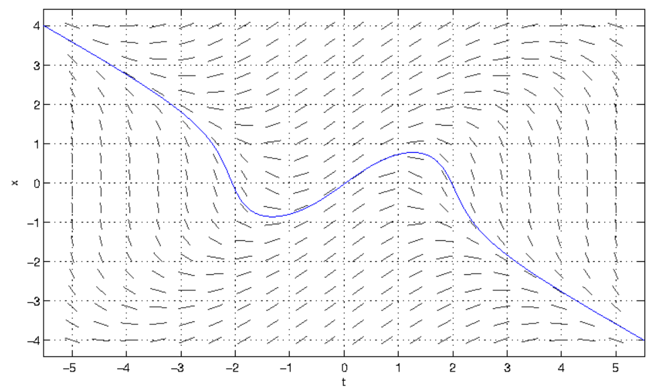
This problem is unusual in that there are many different equivalent solutions, i.e. there are many constants that will work. You merely need to find one constant that works to make this equation true.

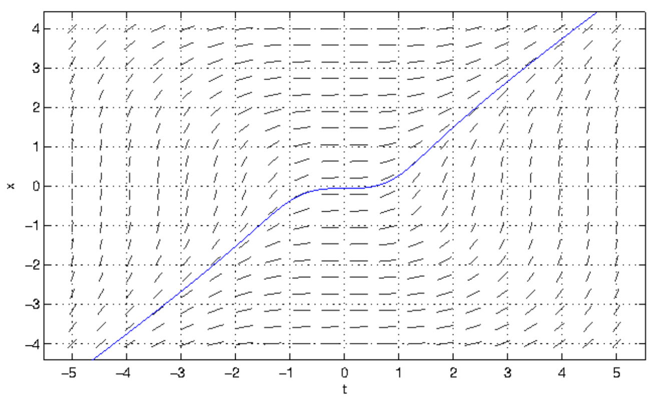
7) The following two direction fields are for the differential equations

$$\text{Equation A:} \quad \frac{dx}{dt} = \frac{t^2}{1+x^2}$$

$$\text{Equation B:} \quad \frac{dx}{dt} = \frac{-t^2}{1+x^2} + 1$$

(2 points) Identify which direction field goes with which equation.




(2 points) Use the correct direction field and solution passing through $x(0) = 0$ to approximate $x(2)$ for **equation A**.

$x(2) \approx$ ________________

8) (5 points extra credit) Let $y' = F(x, y)$ be an ordinary differential equation where $F(x, y)$ is continuous and differentiable with respect to $x$ and $y$ for all $x$ and $y$. Let $y_1(x)$ and $y_2(x)$ be two solutions to this ODE. If $y_1(0) > y_2(0)$ Is it possible that $y_1(1) < y_2(1)$? Why or why not?