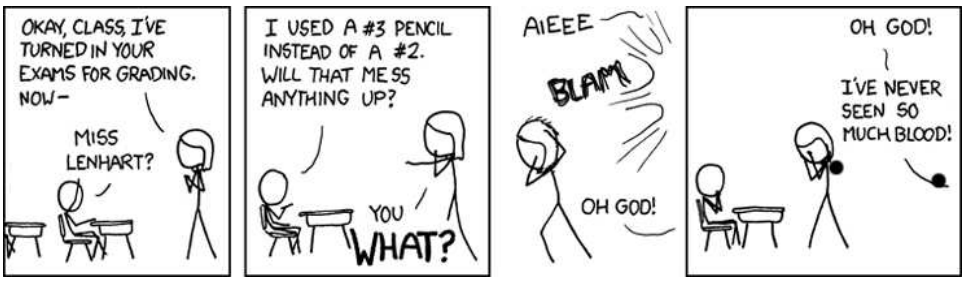
Math 308 Section 517 Exam 2 – Jean Marie Linhart Spring 2009

"An Aggie does not lie, cheat, or steal or tolerate those who do"

On my honor as an Aggie, I have neither given nor received unauthorized aid on this exam.

Printed name:_____

Signature:_____



xkcd.com

- You may use your 4x6 inch notecard for this exam. You must hand it in with your exam.
- You may not use any other notes, a calculator, or your book.
- Your cellphone must be turned off and put away during this exam!
- You may not collaborate with your neighbors on this exam.
- You must show all appropriate work to receive credit, especially partial credit.
- If you use a formula, WRITE IT DOWN.
- The instructor will provide additional scratch paper if needed.
- Read each question carefully.
- GOOD LUCK!!!!!!!

If the IRS had discovered the quadratic formula ...

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Who Can Use Form QF?

You can use Form QF if all of the following apply.

- You need to solve an equation of the form $Ax^2 + Bx + C = 0$.
- A is not equal to zero.

Form QF

1	Enter A here. If line 1 is zero, stop. You cannot use Form QF ...	1	
2	Enter B here ...	2	
3	Enter C here ...	3	
4a	Do you have evidence to support your values of A, B, and C? ...	☐ Yes ☐ No	
b	If "Yes," is the evidence written? ...	☐ Yes ☐ No	
5	Multiply line 1 by 2 ...	5	
6	Divide line 2 by line 5 ...	6	
7	Multiply line 6 by -1 ...	7	
8	Multiply line 3 by line 5 ...	8	
9	Amount from line 2 ...	9	
10	Multiply line 2 by line 9 ...	10	
11	Multiply line 8 by 2 ...	11	
12	Subtract line 11 from line 10. If line 11 is more than line 10, leave blank and fill out Negative Discriminant Worksheet ...	12	
13	If amount on line 12 is zero, enter amount from line 7 on line 15, write "Dbl Rt" in space to left of line 15, and leave line 16 blank. Otherwise, take square root of amount on line 12. Check if square root is from: **a** ☐ Square root tables **b** ☐ Calculator ...	13	
14	Divide line 13 by line 5 ...	14	
15	Root 1: Add lines 7 and 14 ...	15	
16	Root 2: Subtract line 14 from line 7 ...	16	

Negative Discriminant Worksheet

1	Amount from Form QF line 5 ...	1	
2	Amount from Form QF line 7 ...	2	
3	Amount from Form QF line 11 ...	3	
4	Amount from Form QF line 10 ...	4	
5	Subtract line 4 from line 3 ...	5	
6	Take square root of line 5. Check if square root is from: **a** ☐ Square root tables **b** ☐ Calculator ...	6	
7	Divide line 6 by line 1 ...	7	
8	Write amount from line 2, a plus sign, amount from line 7, and the letter "i". Enter here and on Form QF line 15 ...	8	
9	Write amount from line 2, a minus sign, amount from line 7, and the letter "i". Enter here and on Form QF line 16 ...	9	

1) (10 points) Find an explicit solution to the following:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3y = 0 \qquad y(0) = 0, \quad y'(0) = -2\sqrt{2}$$

Hint: Use the quadratic formula.

2) (10 points) Find the general solution of the Cauchy-Euler equation

$$x^2y'' - 3xy' + 4y = 0$$

(10 points) Now use your work from above and find the general solution for

$$x^2y'' - 3xy' + 4y = x^2$$

3) (8 points) According to the existence and uniqueness theorem for 2nd order linear ODEs, on what intervals might the following ODE have unique solutions?

$$(x^2 - 3x)y'' + 2xy' - y = x^2$$

4) (5 points) **YOU DO NOT HAVE TO SOLVE THIS.** If you were using the method of undetermined coefficients to solve this equation, what form of a particular solution would you attempt (guess) and **why**?

$$y'' + 2y' + y = e^{-x}$$

5) (3 points) Use one step of the Euler method to calculate an approximation to $y(1.1)$ for the IVP

$$y' = 2x^2 - xy, \qquad y(1) = 2$$

(4 points) Use one step of the Improved Euler Method to calculate an approximation to $y(1.1)$ for the IVP above.

6) (18 points) Find the solution to the IVP

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 4y = \frac{34}{3}\cos(x) \qquad y(0) = 1, \quad y'(0) = 1$$

7) (15 points) Find the general solution to

$$y'' + 4y' + 4y = e^{-2x} \ln(x)$$

8) (5 points) Show the Laplace transform of $f(t) = \sin(bt)$ is

$$\mathcal{L}\{f\}(s) = \frac{b}{s^2 + b^2}$$

by using the integral definition of the Laplace transform and performing the integral. You may use the integration formula

$$\int e^{cx} \sin(ax) dx = \frac{1}{a^2 + c^2} e^{cx} \left[c \sin(ax) - a \cos(ax)\right] + C$$

rather than integrating by parts twice!

(2 points) For what values of s does $\mathcal{L}\{f\}(s)$ exist for $f(t) = \sin(bt)$?

(5 points) Use the integral definition of Laplace transform to show that if $\mathcal{L}\{g(t)\}(s) = G(s)$ then

$$\mathcal{L}\{e^{at} g(t)\}(s) = G(s - a)$$

(5 points) What is

$$\mathcal{L}\{7e^{3t} \sin(2t) - 3e^{4t}\}?$$

You do not need to use the integral definition of the Laplace transform, but you must show your work!

9) Extra credit. Choose and do **at most one** of the following.

a. (5 points extra credit) The variation of parameters method assumes you have $y_1(x)$ and $y_2(x)$ which are linearly independent solutions to the homogeneous equation

$$L[y] = y'' + p(x)y' + q(x)y = 0$$

then in order to solve

$$L[y] = y'' + p(x)y' + q(x)y = g(x)$$

you attempt a solution of the form $v_1(x)y_1(x) + v_2(x)y_2(x) = y_p(x)$ with the restriction that $v_1'(x)y_1(x) + v_2'(x)y_2(x) = 0$. Using this and the differential equation above, derive the two formulas for $v_1(x)$ and $v_2(x)$.

or

b. (3 points extra credit) Return to your solution to problem 1, which was to find a solution to the IVP

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3y = 0 \qquad y(0) = 0, \quad y'(0) = -2\sqrt{2}$$

Would it make sense for this equation to represent a mass-spring oscillator in the real world? Why or why not?