Week in Review # 10
Sections 10.1, 10.2(some)

1. A cup of coffee contains about 250 mg of caffeine. Caffeine is metabolized and leaves the body at a continuous rate of 21% every hour. Write a differential equation that measures the amount of caffeine, A, in the body as a function of the number of hours, x, since the coffee was consumed.

A = # of mg. of caffeine

Rate = Rate in − Rate out

A' = Rate in mg/hr

$A' = 0 - .21A$

$A' = -.21A$

2. Dead leaves accumulate on the ground in a forest at a rate of 4 grams per square centimeter per year. At the same time, these leaves decompose at a continuous rate of 70% per year. Write a differential equation for the total quantity of dead leaves(per square centimeter) at time x.

L = amt of leaves in grams per sq. cm.

L' = Rate grams per sq. cm / yr.

$L' = 4 - .7L$

Title: Apr 23-2:47 PM (1 of 9)

3. A patient is given a drug intravenously at a rate of 43.2 mg/hour. The rate at which the drug leaves the body is proportional to the quantity present. When there are 100mg of the drug in a patient, then the rate that the drug is leaving is 8.2 mg/hour. Write a differential equation that measures the amount of the drug in the body as a function of the number of hours since it was started.

A = amt of drug in mg

A' = rate in mg/hr.

rate in − Rate out.

$A' = 43.2 - .082A$

Rate out = KA

$8.2 = K(100)$

$.082 = K$

Rate out = $.082A$

4. A population of fish was modeled by the following differential equation where P has units of millions of fish, and P' has units of millions of fish per year.

$$P' = .45P - 18$$

Assuming the the population started at 85 million fish and that time is measured from this starting value.

(a) Use the information provided to approximate the population of the fish at the indicated values of t.

t	P (millions)
0	85
1	105.25
2	134.6125
3	177.1881

$P' = .45(85) - 18$

$= 20.25$ mill/yr

new pop $= 85 + 1(20.25)$

$= 105.25$ million

$P' = .45(105.25) - 18$

$= 29.3625$

new pop. $= 105.25 + 1(29.3625)$

(b) Use the information provided to approximate the population of the fish at the indicated values of t.

$$P' = .45P - 18$$

t	P (millions)
0	85
0.5	95.125
1	107.5281
1.5	122.7219
2	141.3343
2.5	164.1345
3	192.0648

$P' = .45(85) - 18$

$= 20.25$

New population $= 85 + (.5)(20.25)$

$= 95.125$

Repeat the method

(c) Show that $P = 40 + Ce^{0.45t}$ is a solution to the differential equation.

$$P' = .45P - 18$$

$P' = .45Ce^{.45t}$

Left side

$.45ce^{.45t}$

equal

Right side

$.45(40 + Ce^{.45t}) - 18$

$.45(40) + .45Ce^{.45t} - 18$

$18 + .45ce^{.45t} - 18$

$.45ce^{.45t}$

So it is a solution

(d) Find the value of C in part (c)

The intial population (at the start ie $t=0$) is 85 million.

$85 = 40 + Ce^{0}$

$85 = 40 + C$ → $C = 45$

(e) Which method, part (a) or part (b), gave a better approximation to the population of fish three years after the start?

$P = 40 + 45e^{.45t}$

at $t = 3$

$P = 40 + 45e^{.45(3)}$

$= 213.5841$ million fish

part (b) gave a better estimate.

5. Is $y = x^3 + 2x + 7$ is a solution to the differential equation $3y - xy' = 4x + 18$?

$y' = 3x^2 + 2$

Left side	Right side
$3(x^3+2x+7) - x(3x^2+2)$	$4x + 18$
$\cancel{3x^3} + 6x + 21 - \cancel{3x^3} - 2x$	
$4x + 21$	

Not equal so it is not a solution.

6. Is $y = 2e^{5x} + 3x$ a solution to the differential equation $y'' - 4y' + 12 = 5y - 15x$?

$y' = 10e^{5x} + 3$

$y'' = 50e^{5x}$

L.S.	R.S.
$50e^{5x} - 4(10e^{5x} + 3) + 12$	$5(2e^{5x} + 3x) - 15x$
$50e^{5x} - 40e^{5x} - 12 + 12$	$10e^{5x} + 15x - 15x$
$10e^{5x}$	$10e^{5x}$

equal so it is a solution.

7. Find the value of k so that $y = x^4 + kx$ is a solution to the differential equation $4y - xy' = 30x$

$$y' = 4x^3 + k$$

$$4(x^4 + kx) - x(4x^3 + k) = 30x$$

$$\cancel{4x^4} + 4kx - \cancel{4x^4} - kx = 30x$$

$$3kx = 30x$$

this implies that $3k = 30$

or $k = 10$

8. Find the values of c and k such that $y = ce^{kx}$ is a solution to the differential equation $5y' = 3y$.

$y' = kce^{kx}$

$5(kce^{kx}) = 3(ce^{kx})$

$5kce^{kx} = 3ce^{kx}$

since e^{kx} is never zero divide both sides by e^{kx}

$5kc = 3c$

Since c is on both sides & is multiplied

Then c can be any #.

for this to work we need

$5k = 3$ or $k = \frac{3}{5}$