1. The graph of \( f \) is given to the right.

   (a) Find the intervals (x-intervals) where \( f \) is decreasing and concave up.

   \( 0 < x < 2 \)

   \( x > 8 \)

   (b) Find the intervals (x-intervals) where \( f \) is increasing and concave down.

   \( 4 < x < 6 \)
Week in Review # 2
Section 1.3 and 1.5

\[
\frac{f(c) - f(4)}{6 - 4} = \frac{4 - 1}{6 - 4} = \frac{3}{2}
\]

(c) On which interval \(4 \leq x \leq 6\) or \(0 \leq x \leq 8\) is the average rate of change the largest?

\[
\frac{f(8) - f(0)}{8 - 0} = \frac{2 - 0}{8 - 0} = \frac{1}{4}
\]

(d) Which of these is the largest?

\[
\frac{f(8) - f(4)}{8 - 4} \quad \text{or} \quad \frac{f(7) - f(3)}{7 - 3}
\]

\[
f(4) = 2 \quad \quad f(7) = 3.4
\]

\[
f(4) = 1 \quad \quad f(3) = -2
\]

\[
\frac{2 - 1}{8 - 4} = \frac{1}{4}
\]

\[
\frac{3.4 - 2}{7 - 3} = \frac{5 - 4}{4}
\]

\text{just slopes (with is bigger)}
2. Sketch a graph that is increasing and concave up for $x < 1$
   decreasing and concave up for $1 < x < 5$
   decreasing and concave down for $x > 5$

3. Compute the average rate of change for $f(x) = 5x - 2x^2 + 7$ from $x = 1$ to $x = 4$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{f(4) - f(1)}{4 - 1} = \frac{-5 - 10}{3} = \frac{-15}{3} = -5$$
4. The figure gives the balancce, in dollars, of a bank account $t$ years after it has been started.

(a) Compute and interpret the average rate of change from $t = 0$ to $t = 2$.

$$\frac{f(2) - f(0)}{2 - 0} = \frac{2000 - 1500}{2 - 0} = \frac{500}{2} = \$250/yr$$

For the first 2 yrs the balance of the account grew on average by $250 each yr.

(b) Compute and interpret the average rate of change from $t = 2$ to $t = 4$.

$$\frac{f(4) - f(2)}{4 - 2} = \frac{6000 - 2000}{2} = \$2000/yr$$

For the 3rd and 4th years (ie from $t = 2$ to $t = 4$) the balance grew on avg by $2000 each year.

(c) Graphically represent your answers for parts (a) and (b)
5. The following graph shows the daily average retail price of regular gasoline in the Dallas area.

(a) What is the average rate of change of gas prices from December 26 to January 16? Interpret this answer.

\[
\frac{\Delta p}{\Delta t} = \frac{2.07 - 2.20}{21} = -\frac{0.00619}{\text{day}}
\]

On average the price per gallon of gas is dropping by \$0.006190 each day from Dec 26 to Jan 16.

(b) What is the average rate of change of gas prices from December 29 to January 8? Interpret this answer.

\[
\frac{2.14 - 2.20}{10} = -0.006
\]

The price per gallon of gas is dropping by \$0.006 each day from Dec 29 to Jan 8.
6. For these exponential formulas give the initial value and the relative growth/decay rate (percent rate of change).

(a) $y = 37(.87)^x$

- Initial = $37$
- $a = .87$
- $1 + r = .87$
- $r = -.13$

**Relative decay rate = 13%**

(b) $y = 100(1.034)^x$

- Initial = $100$
- $a = 1.034$
- $1 + r = 1.034$
- $r = .034$

**Relative rate of growth = 3.4%**
7. Find the exponential formula for the data sets.

(a) \[
\begin{array}{c|c|c}
 x & 2 & 5 \\
\hline
 y & 50 & 6.25 \\
\end{array}
\]

\[y = 200 (0.5)^x\]

\[50 = P_0 a^2\]

\[6.25 = \frac{50}{a^2}\]

\[6.25 = 50 a^3\]

\[6.25 = 50 a^3\]

\[125 = a^3\]

\[a = (125)^{\frac{1}{3}} = 5\]

(b) \[
\begin{array}{c|c|c}
 x & 3 & 5 \\
\hline
 y & 1728 & 2488.32 \\
\end{array}
\]

\[y = 1000 (1.2)^x\]

\[2000 = P_0 a^2\]

\[6.25 = \frac{2000}{a^2}\]

\[6.25 = 2000 a^3\]

\[6.25 = 2000 a^3\]

\[125 = a^3\]

\[a = (125)^{\frac{1}{3}} = 5\]
8. Give a possible formula for these statements.

(a) The city’s population increased by 12% each year.

\[ y = P_0 (1.12)^x \]
\[ \text{\textit{\textup{\textcolor{red}{\text{\textup{x}}}}}} \text{ is years} \]
\[ \text{\textit{\textup{\textcolor{red}{\text{\textup{\textcolor{red}{\text{\textup{\textcolor{red}{\text{\textup{x}}}}} \text{ initial size}}}}}}} \]

(b) The city’s population increased by 200,000 each year.

\[ y = mx + b \]
\[ y = 200000x + P_0 \]
\[ \text{\textit{\textup{\textcolor{red}{\text{\textup{\textcolor{red}{\text{\textup{\textcolor{red}{\text{\textup{x}}}}} \text{ initial when } x=0}}}}} \]

(c) A person is given a dose of a drug and the drug disappears (used by the body) by 4% per minute.

\[ y = P_0 (0.96)^x \]
\[ \text{\textit{\textup{\textcolor{red}{\text{\textup{x}}}}} \text{ in minutes} \]
\[ 1 + r \quad r = -0.04 \]

(d) The lollipops at a pediatrician office are decreasing at a rate of 135 per day.

\[ y = mx + b \]
\[ y = -135x + P_0 \]
\[ \text{\textit{\textup{\textcolor{red}{\text{\textup{\textcolor{red}{\text{\textup{\textcolor{red}{\text{\textup{x}}}}} \text{ initial amt.}}}}} \]
9. You are told that the function $f(x)$ is an exponential function and that $f(2) = 300$. If the function has an average rate of change of 600 from $x = 2$ to $x = 5$, find an exponential formula for this function.

\[
\text{Avg. Rate of Change} \quad \frac{f(5) - f(2)}{5 - 2} = 600
\]

\[
\frac{f(5) - 300}{3} = 600
\]

\[
f(5) - 300 = (600) \cdot 3
\]

\[
f(5) = (600) \cdot 3 + 300 = 2100
\]

\[
x = 2 \quad \frac{5}{y = 300} \quad \frac{2100}{\text{Exp. Reg. 5}}
\]

\[
y = 81.9827 \cdot (1.9129 \cdot 3^{1.5})^x
\]
10. A patient is given a 75mg dose of a drug. This drug leaves the body at a rate of 8.25% per hour.

(a) How much of the drug is in the body after 3 hours?

\[ y = 75 \left(1 - 0.0825\right)^x = 75 \left(0.9175\right)^x \]

Plug in 3 for \(x\)

\[ 57.92679 \text{ mg} \]

(b) How much of the drug is in the body after 12 hours?

Plug in 12 for \(x\)

\[ 26.68911 \text{ mg} \]
11. A pesticide has a half life of 7 days. A crop is sprayed with this pesticide.

(a) Find the percentage of the chemical still on the plants after 3 days.

\[ y = 100 \left(0.90572366^x\right) \]

plug in 3 for x

74.2997%

(b) The FDA will not approve a pesticide for use on commercial crops if that pesticide has a relative rate of decay that is less than 12% per day. Is this pesticide approved for use on commercial crops by the FDA? justify your answer.

\[ r = -0.09427634 \]

rate of decay is 9.427634%

(c) How long will it take until less than 25% of the pesticide is still on the plants?

after 14 days.