

**Week in Review # 3**  
**Section 1.6, 1.7, and 1.8**

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1. Simplify with logarithm rules.

$$\begin{aligned} \text{(a) } \ln(x^4 z^5) &= \ln(x^4) + \ln(z^5) \\ &= 4 \ln(x) + 5 \ln(z) \end{aligned}$$

$$\begin{aligned} \text{(b) } \ln\left(\frac{e^{3x}}{x^5}\right) &= \ln(e^{3x}) - \ln(x^5) \\ &= 3x - 5 \ln(x) \end{aligned}$$

2. Solve for x.

(a)  $J = 5 * 8^x$

$$\ln(J) = \ln(5 * 8^x)$$

$$\ln(J) = \ln(5) + \ln(8^x)$$

$$\ln(J) - \ln(5) = x \ln(8)$$

$$x = \frac{\ln(J) - \ln(5)}{\ln(8)}$$

Method #2

$$\frac{J}{5} = 8^x$$

$$\ln\left(\frac{J}{5}\right) = \ln(8^x)$$

$$\ln\left(\frac{J}{5}\right) = x \ln(8)$$

$$x = \frac{\ln(J/5)}{\ln(8)}$$

(b)  $8 * 3^x = 2 * 7^x$

$$\ln(8 * 3^x) = \ln(2 * 7^x)$$

$$\ln(8) + \ln(3^x) = \ln(2) + \ln(7^x)$$

$$\ln(8) + x \ln(3) = \ln(2) + x \ln(7)$$

$$\ln(8) - \ln(2) = x \ln(7) - x \ln(3)$$

$$\ln(8) - \ln(2) = x [\ln(7) - \ln(3)]$$

$$\frac{(\ln(8) - \ln(2))}{(\ln(7) - \ln(3))} = x$$

exact.

$$= 1.6361$$

↑ approx.

$$a = 1+r$$

3. For each of these formulas

I) Convert them into the form  $y = P_0 a^t$  or  $y = P_0 e^{kt}$

II) Give the relative rate of growth/decay.

III) Give the continuous rate of growth/decay.

$$P_0 e^{kt}$$

$$a = e^k$$

$$\ln a = k$$

(a)  $y = 35(1.25)^t$

I)  $y = 35 e^{.22314355t}$

II) growth 25%

III) growth 22.314355%

(b)  $y = 27e^{-.127t}$

I)  $y = 27 (.88073367)^t$

II) decay 11.92663274%

III) decay 12.7%

4. When solving for the yearly continuous rate of decay for a substance a student rounded the answer to two decimal places and got the answer of 2%.

(a) If the correct rate of decay was 1.505%, find the half-life of the substance.

$$50 = 100 e^{-.01505 t}$$

$$\frac{1}{2} = e^{-.01505 t}$$

$$\ln(.5) = -.01505 t$$

$$\frac{\ln(.5)}{-.01505} = t \approx 46.05629 \text{ yrs}$$

(b) If the correct rate of decay was 2.4%, find the half-life of the substance.

$$50 = 100 e^{-.024 t}$$

⋮

$$\frac{\ln(.5)}{-.024} = t \approx 28.8811 \text{ years}$$

(c) How does the half-life of the student's rate of decay compare with the answers in parts (a) and (b).

$$50 = 100 e^{-.02 t}$$

⋮

$$\frac{\ln(.5)}{-.02} = t \approx 34.657355 \text{ years}$$

Student's answer has a very different half-life.

5. A bank account was started with \$600. Two years later the account had \$850.

(a) What is the continuous interest rate for the account?

$$B = Pe^{rt}$$

$$850 = 600e^{2r}$$

$$\frac{850}{600} = e^{2r}$$

$$\ln\left(\frac{850}{600}\right) = 2r$$

$$r = \frac{\ln\left(\frac{850}{600}\right)}{2}$$

$$r = .174153347$$

$$17.4153347\%$$

(b) How long will it take for the account to triple?

$$3 = 1e^{.174153347t}$$

$$\ln(3) = .174153347t$$

$$\frac{\ln(3)}{.174153347} = t \approx 6.30834 \text{ yrs}$$

6. You have been offered a payment of \$5000 in 4 years and a payment of \$8000 in 7 years. What is the present value of this offer if the interest rate is

(a) 6.25% compounded annually.

$$B = P(1+r)^t \rightarrow P = B(1+r)^{-t}$$

$$5000(1+0.0625)^{-4} + 8000(1+0.0625)^{-7}$$

$$\$9,156.77$$

(b) 6.25% compounded continuously.

$$B = Pe^{rt} \rightarrow P = Be^{-rt}$$

$$5000e^{-0.0625(4)} + 8000e^{-0.0625(7)}$$

$$\$9,059.19$$

7. For the functions  $f(x) = \sqrt{x+5}$  and  $g(x) = 2x^2 + 3$  find

$$(a) \quad g(\underline{f(7)}) = g(\sqrt{12}) = 2(\sqrt{12})^2 + 3$$

$$f(7) = \sqrt{12} \quad = 2(12) + 3 \\ = \boxed{27}$$

$$(b) \quad f(\underline{g(2)}) = f(11) = \sqrt{16}$$

$$g(2) = 11 \quad = \boxed{4}$$

$$(c) \quad g(3 + \underline{g(1)}) = g(3+5) = g(8)$$

$$g(1) = 5 \quad = 2(8)^2 + 3 \\ = \boxed{131}$$

$$(d) \quad f(\underline{g(x)}) = f(2x^2+3)$$

$$= \sqrt{(2x^2+3)+5} \\ = \boxed{\sqrt{2x^2+8}}$$

$$(e) \quad g(\underline{f(x)}) = g(\sqrt{x+5}) = 2(\sqrt{x+5})^2 + 3$$

$$= 2(x+5) + 3$$

$$= 2x + 10 + 3$$

$$= 2x + 13$$

8. Find two function  $f(x)$  and  $g(x)$  such that  $h(x) = \underline{f(g(x))}$

$$\underline{h(x) = 5 \ln(3x^2 + 1)}$$

More than one solution

$$g(x) = 3x^2$$

$$f(x) = 5 \ln(x+1)$$

$$f(g(x)) = f(3x^2)$$

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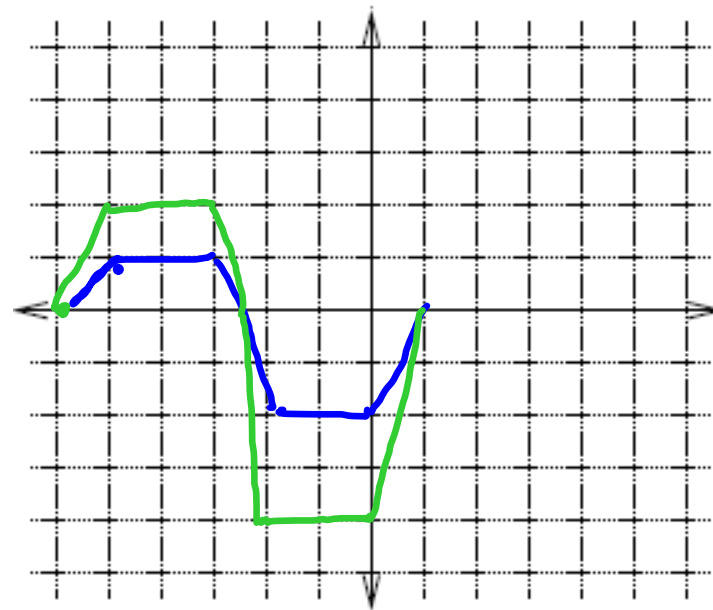
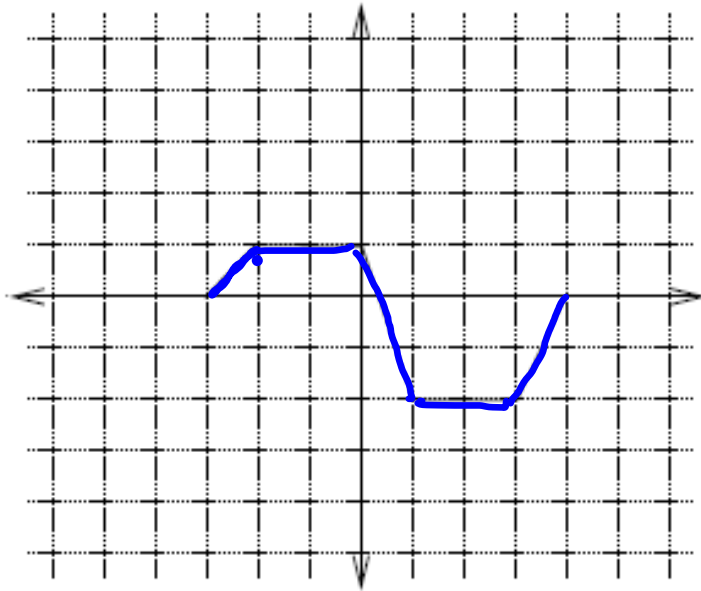
$$g(x) = 3x^2 + 1$$

$$f(x) = 5 \ln(x)$$

9. The graph of  $f(x)$  is given. Use it to sketch the following.

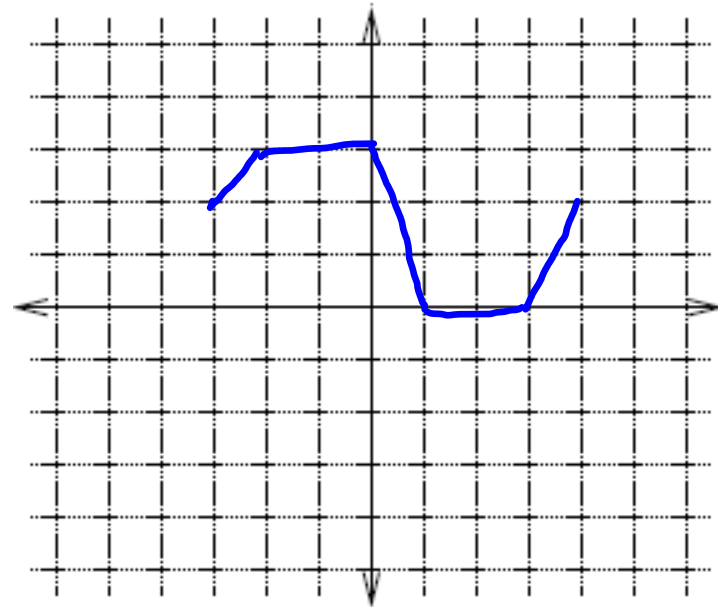
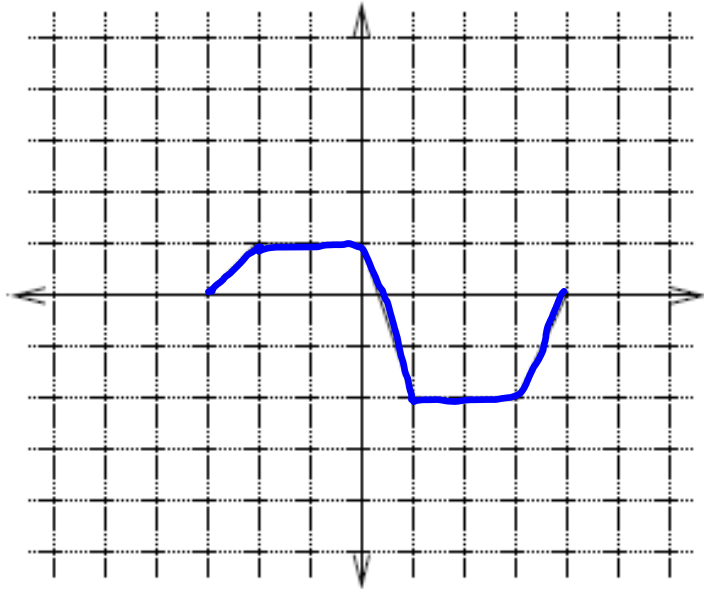
$$2f(x+3)$$

(a)  $f(x+3)$



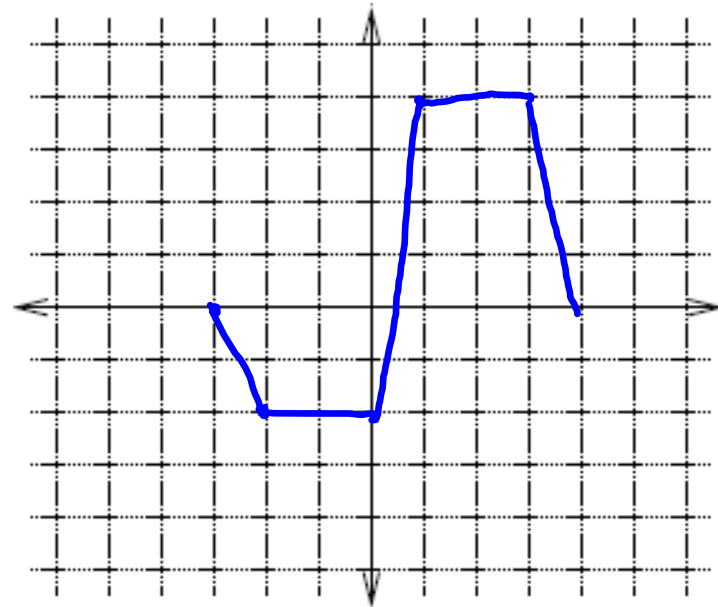
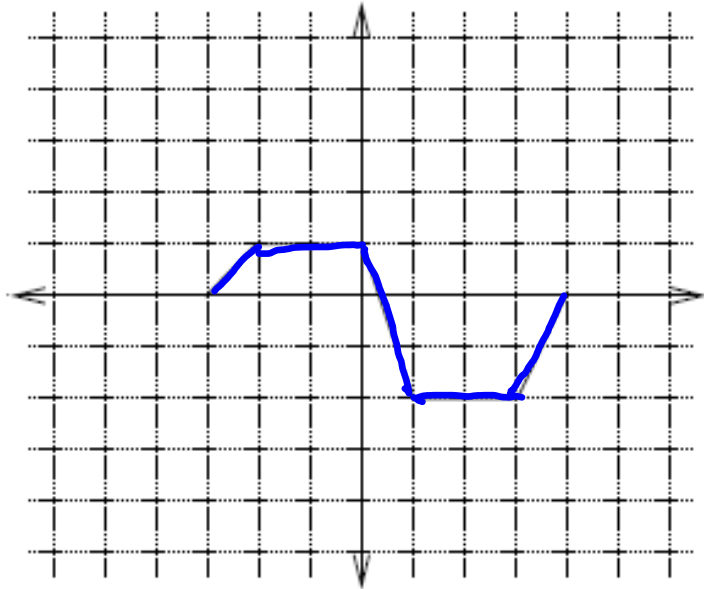
9. The graph of  $f(x)$  is given. Use it to sketch the following.

(b)  $f(x)+2$



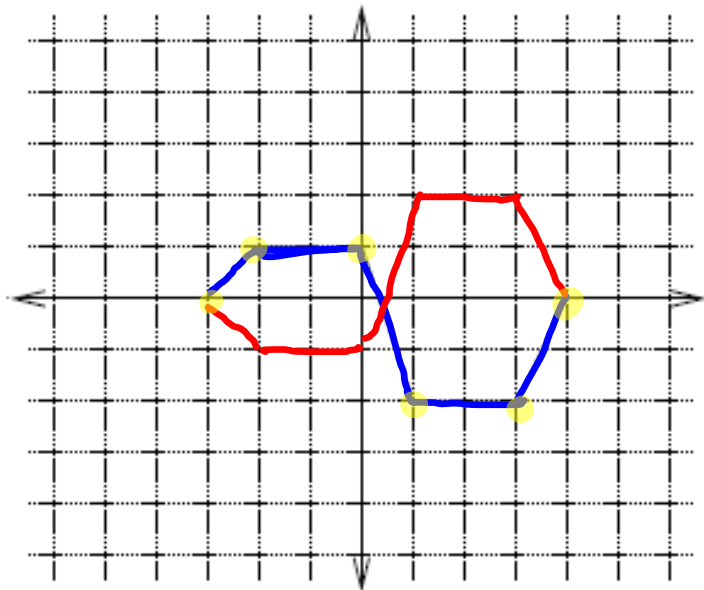
9. The graph of  $f(x)$  is given. Use it to sketch the following.

(c)  $-2f(x)$



9. The graph of  $f(x)$  is given. Use it to sketch the following.

$-f(x)$



(d)  $2 - f(x) = -f(x) + 2$

