1. Fill in the blanks with the relationships between $f(x)$, $f'(x)$, and $f''(x)$.

$f'(x) > 0$ means that f(x) is inc.

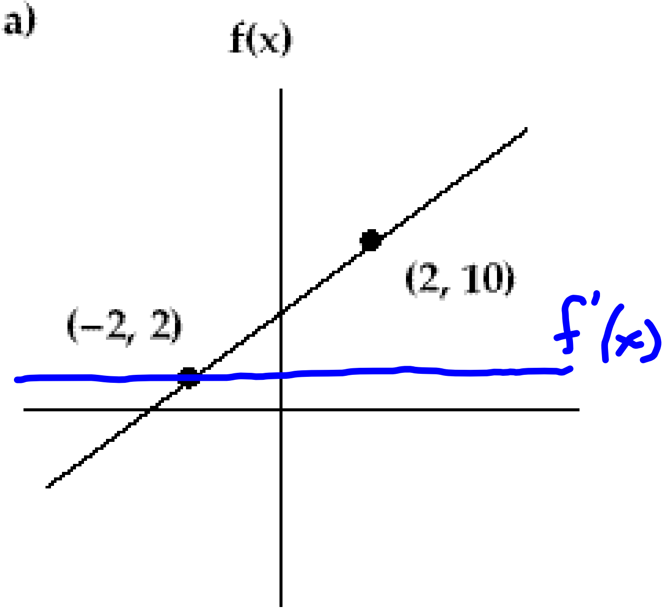
$f'(x) < 0$ means that f(x) is dec.

$f''(x) > 0$ means that f'(x) is inc. and f(x) is concave up

$f''(x) < 0$ means that f'(x) is dec. and f(x) is concave down.

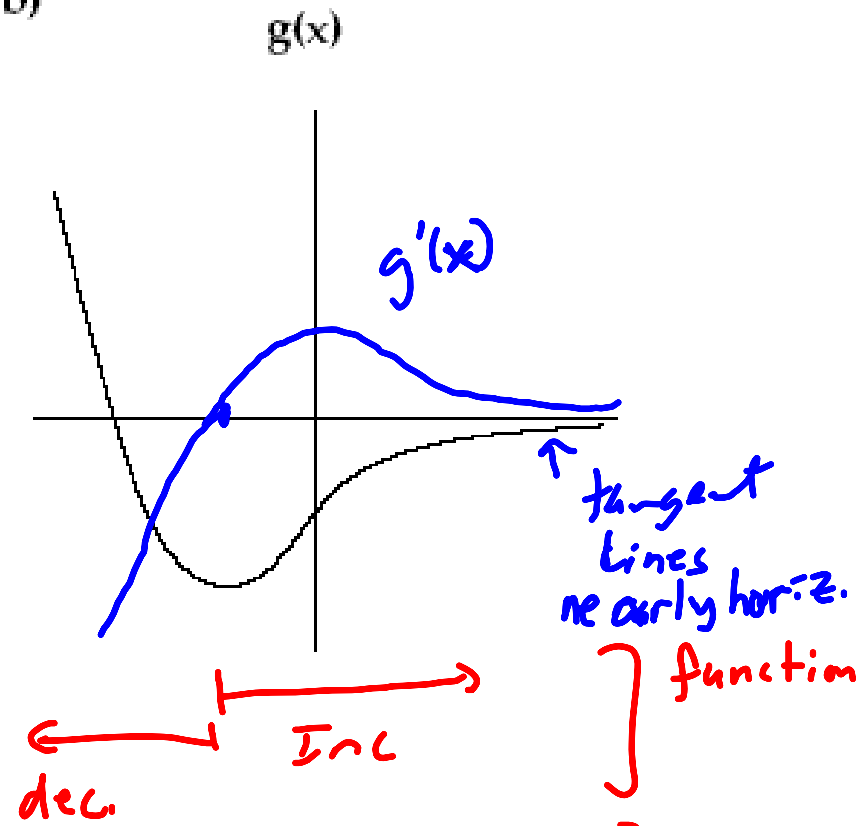
2. Sketch the graphs of the derivatives of each of these functions.

(a)



$m = \frac{8}{4} = 2$

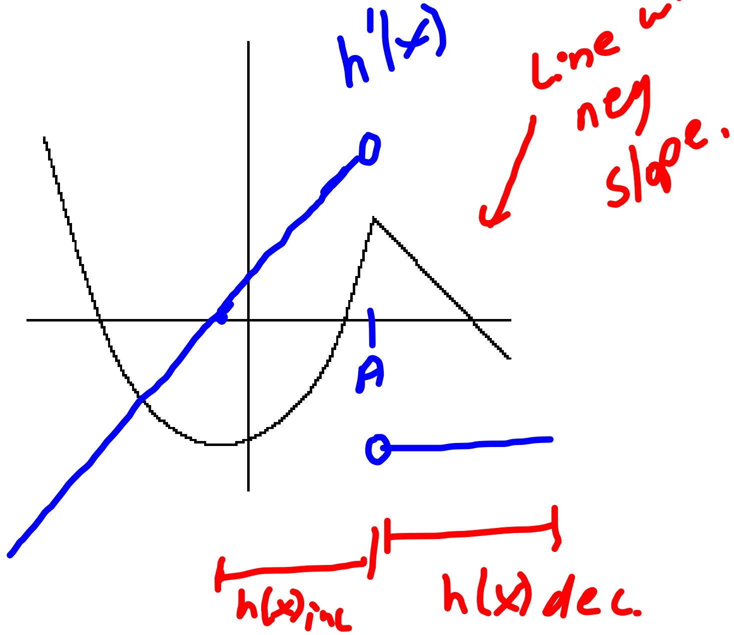
(b)



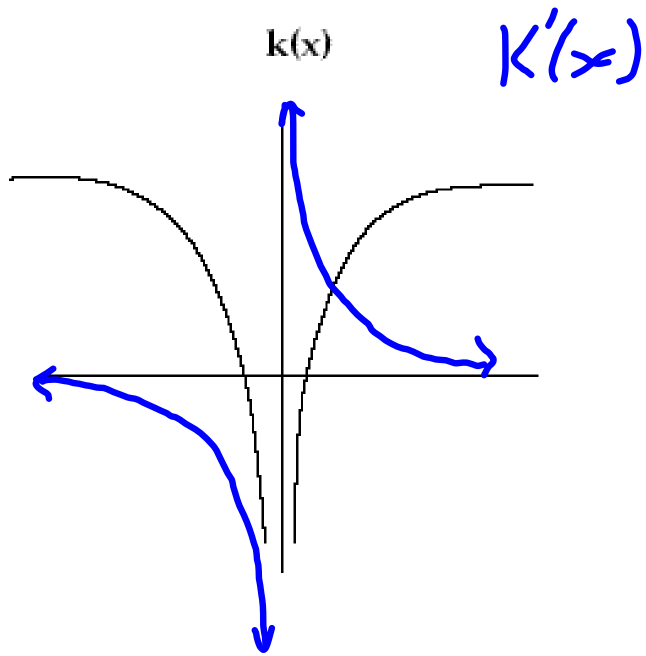
} function.

neg pos. } deriv.

(c)



(d)



at x=A sharp point.

Note: A function doesn't have a deriv. at a sharp point.

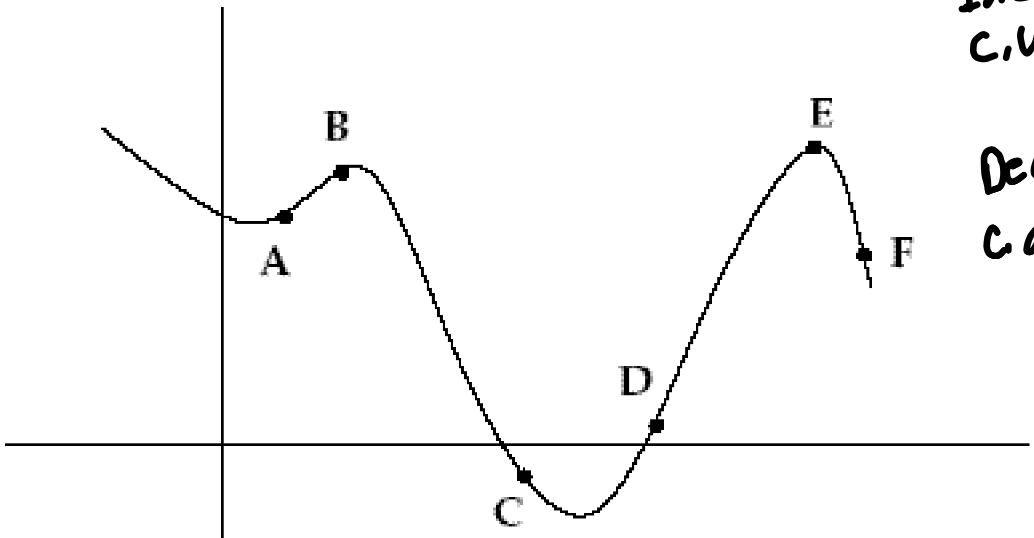
3. Here is the graph of the function $f(x)$.

Look at tangent lines.

(a) Arrange the derivatives at the given points from smallest to largest.

smallest F, C, E, B, A, D Largest

(b) At which points does $f'(x)$ and $f''(x)$ have the same sign?



Inc. c.u. pos: A, D

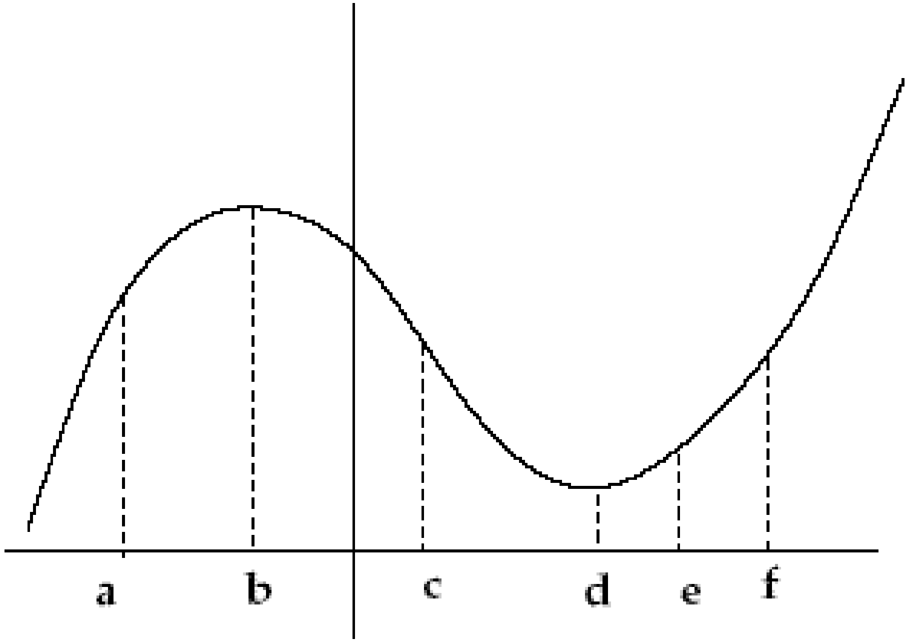
Dec c.d. neg: F

pos: D, A, B

neg: C, F

zero: E

4. Match the points with the derivatives.



~~a~~
e
f

x	d	e	B	c	a	f
$f'(x)$	0	1	0	-2	2	2
$f''(x)$	2	3	-2	0	-4	4

f
B/d
B/d.

time $\quad$ deg.

5. Suppose $H = f(t)$ is the time, in minutes, that it takes a deep fryer to heat up to t^0F.

   (a) What are the units of $f'(t)$ and what is the sign of $f'(t)$?

   $$\frac{df}{dt} = \frac{\text{min}}{{}^\circ F}$$

   pos.

   (b) What is the meaning of $f(350) = 15$?

   It takes 15 min to heat to 350°F.

   (c) what is the meaning of $f'(350) = 0.25$?

   at the 350°F to go up 1°F it would take approx 0.25 min.

   (d) Estimate the time for the deep fryer to heat up to 375°F.

   $$\begin{array}{r} 375 \\ -\ 350 \\ \hline 25\,^\circ F \end{array}$$

   $$f(375) \approx \underbrace{15}_{f(350)} + \underbrace{(0.25)}_{f'(350)}\underbrace{(25)}_{(375-350)}$$

   $$\approx 21.25 \text{ min.}$$

yrs. $P(t)$ is in \$

6. Suppose $P(t)$ is the monthly payment, in dollars, on a mortgage which will take t years to pay off.

   (a) What are the units of $P'(t)$ and the sign of $p'(t)$?

   $\frac{dP}{dt} = \$/\text{yr.}$ neg.

   (b) What is the practical meaning of $P'(t)$?

   how the monthly payments are decreasing for certain years.

   ---

   rate of change of the monthly payment as time increases.

7. Suppose $g(20) = 125$ and $g'(20) = -8$. Estimate $g(18)$, $g(25)$, and $g(31)$.

$$g(18) \approx \underbrace{125}_{g(20)} + \underbrace{(-8)}_{g'(20)} \underbrace{(-2)}_{(18-20)} = 125 + 16 = 141$$

$$g(25) \approx g(20) + g'(20)(25-20)$$

$$125 + (-8)(5) = 125 - 40 = 85$$

$$g(31) \approx \underbrace{125}_{g(20)} + \underbrace{(-8)}_{g'(20)} \underbrace{(11)}_{(31-20)} = 125 - 88$$

$$= 37$$

8. If $f(3) = 20$, $f'(3) = 2$ and $\overset{f''}{\cancel{f}}(x) < 0$ for $x \geq 3$, what can you say about the value of $f(7)$?

concave down.

find coord. of the point (7,28)

for tangent Line.

$y - y_1 = m(x - x_1)$

$y - 20 = 2(x - 3)$

eq. of tangent Line

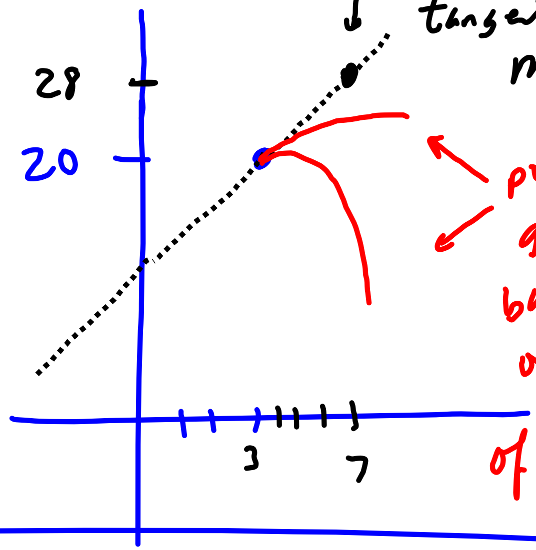
plug in a 7 for x

$y - 20 = 2(7 - 3)$

$y = 20 + 8$

$y = 28$

tangent Line $m = 2$



possible graph based on the info. of $f''(x)$

If there was no concavity the function would go through the point (7, 28).

Since $f(x)$ is concave down for $x \geq 3$ we know $f(7) < 28$.

9. The temperature inside a house was given by $f(t)$ in °F. At 1pm, the temperature was 70°F. The first derivative, $f'(t)$ decreased until reaching a value of 1°F/hour at 1pm, then increased for the rest of the day. sketch a graph of the temperature inside the house during this time period.

unit of $f'(t)$ $\frac{°F}{hr}$ so units of t are hrs.

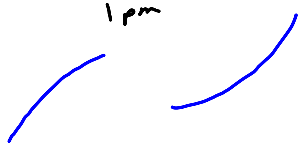
$f(x)$ is concave down, $f'(x)$ dec.	1pm	$f(x)$ concave up, $f'(x)$ inc.

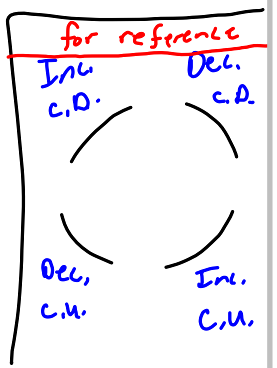
says that $f'(x) \geq 1$ °F/hr. all day long.

Meaning $f'(x)$ was pos. all day long.

Says $f(x)$ is increasing all day long.

$f(x)$	c.d. Inc.	1pm	c.u. Inc.





Sketch of $f(x)$

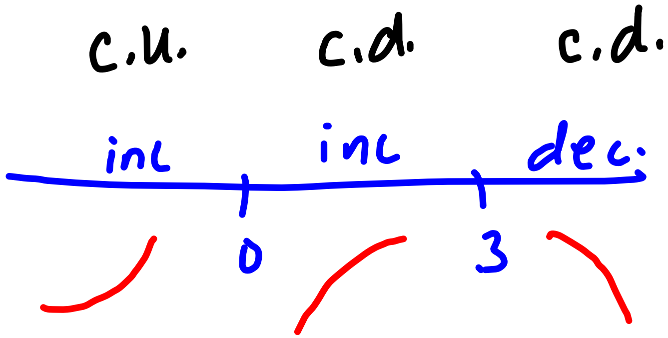
10. Sketch a graph of a function that meets these conditions.

- $f(x)$ is positive for $x < 0$
- $f'(x) > 0$ for $x < 3$
- $f'(x) < 0$ for $x > 3$
- $f''(x) < 0$ for $x > 0$
- $f''(x) > 0$ for $x < 0$
- $f'(3) = 0$

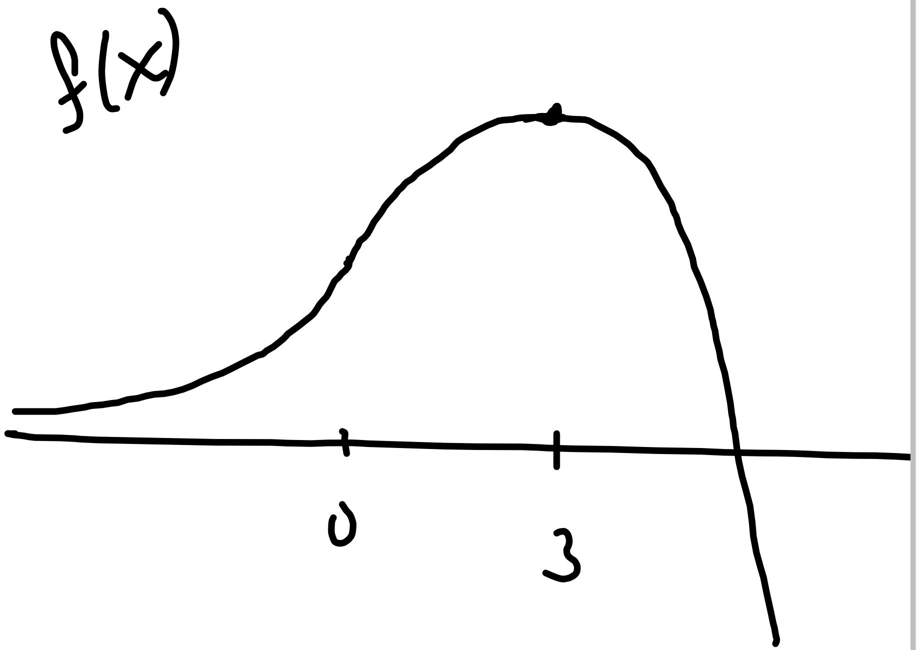
correction.

f''(x) info.

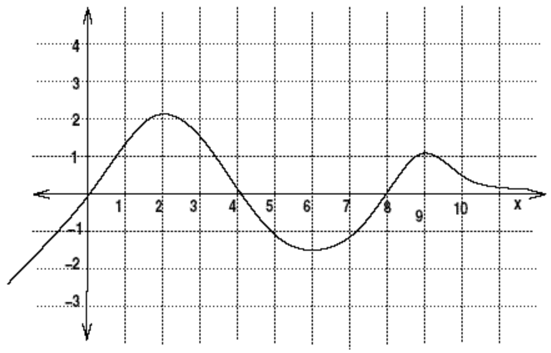
f'(x) info.



above x-axis for $x < 0$



11. Here is the graph of $f'(x)$.



(a) On what intervals is $f(x)$ increasing? $0 < x < 4$, $x > 8$

$f'(x)$ pos

(b) On what intervals is $f(x)$ decreasing? $x < 0$, $4 < x < 8$

$f'(x)$ neg.

(c) On what intervals is $f(x)$ concave up? $x < 2$, $6 < x < 9$

$f'(x)$ inc.

(d) On what intervals is $f(x)$ concave down? $2 < x < 6$, $x > 9$

$f'(x)$ dec.

(e) Use the above information to sketch a graph of $f(x)$.