1. Fill in the blanks with the relationships between $f(x)$, $f'(x)$, and $f''(x)$.

- $f'(x) > 0$ means that $f(x)$ is inc.
- $f'(x) < 0$ means that $f(x)$ is dec.
- $f''(x) > 0$ means that $f'(x)$ is inc. and $f(x)$ is concave up.
- $f''(x) < 0$ means that $f'(x)$ is dec. and $f(x)$ is concave down.
2. Sketch the graphs of the derivatives of each of these functions.

(a) \( f(x) \)

\[ m = \frac{8}{4} = 2 \]

(b) \( g(x) \)

\[ g'(x) \]

Inc \quad Dec

Pos \quad Neg

Target lines nearly horizontal function deriv.
\[ x = \text{a sharp point.} \]

Note: A function doesn't have a deriv. at a sharp point.
3. Here is the graph of the function $f(x)$.

(a) Arrange the derivatives at the given points from smallest to largest.

- $F$, $C$, $E$, $B$, $A$, $D$

(b) At which points does $f'(x)$ and $f''(x)$ have the same sign?

- **Inc.** $\overline{\text{pos}}: A, D$
- **Dec.** $\overline{\text{neg}}: F$

- **Pos** $D, A, B$
- **Neg** $C, F$
- **Zero** $E$
4. Match the points with the derivatives.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>$e$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'(x)$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$f''(x)$</td>
<td>2</td>
<td>3</td>
<td>-2</td>
<td>0</td>
<td>-4</td>
<td>4</td>
</tr>
</tbody>
</table>
5. Suppose $H = f(t)$ is the time, in minutes, that it takes a deep fryer to heat up to $t^\circ F$.

(a) What are the units of $f'(t)$ and what is the sign of $f'(t)$?

$$\frac{df}{dt} = \frac{\text{min}}{\text{OF}}$$

(b) What is the meaning of $f(350) = 15$?

It takes 15 min to heat to 350°F.

(c) What is the meaning of $f'(350) = 0.25$?

At the 350°F to go up 1°F it would take approx 0.25 min.

(d) Estimate the time for the deep fryer to heat up to 375°F.

$$f(350) \approx 15 + (0.25)(25) \approx 21.25 \text{ min.}$$
6. Suppose $P(t)$ is the monthly payment, in dollars, on a mortgage which will take $t$ years to pay off.

(a) What are the units of $P'(t)$ and the sign of $p'(t)$?

\[
\frac{dp}{dt} = \$ \text{yr.} \quad \text{neg.}
\]

(b) What is the practical meaning of $P'(t)$?

how the monthly payments are decreasing for certain years.

rated change of the monthly payment as time increases.
7. Suppose $g(20) = 125$ and $g'(20) = -8$. Estimate $g(18)$, $g(25)$, and $g(31)$.

$$g(20) \approx g(20) + g'(20)(18-20)$$

$$g(18) \approx 125 + (-8)(-2) = 125 + 16 = 141$$

$$g(25) \approx g(20) + g'(20)(25-20)$$

$$g(25) \approx 125 + (-8)(5) = 125 - 40 = 85$$

$$g(31) \approx g(20) + g'(20)(31-20)$$

$$g(31) \approx 125 + (-8)(11) = 125 - 88 = 37$$
8. If \( f(3) = 20, f'(3) = 2 \) and \( f''(x) < 0 \) for \( x \geq 3 \), what can you say about the value of \( f(7) \)?

```
for tangent line.
\[
y - y_1 = m(x - x_1)
\]
\[
y - 20 = 2(x - 3)
\]
Slope of tangent line
plug in 7 for \( x \)
\[
y - 20 = 2(7 - 3)
\]
\[
y = 20 + 8
y = 28
\]
If there was no concavity
the function would go through the point \((7, 28)\).
Since \( f''(x) \) is concave down for \( x \geq 3 \)
we know \( f(7) < 28 \).```

Possible graph based on the info. of \( f''(x) \)

Find coord. of point \((7, 28)\)

Tangent line \( m = 2 \)
9. The temperature inside a house was given by $f(t)$ in °F. At 1pm, the temperature was 70°F. The first derivative, $f'(t)$, decreased until reaching a of 1°F/hour at 1pm, then increased for the rest of the day. Sketch a graph of the temperature inside the house during this time period.

The units of $f'(t)$ are °F/hr. So units of $t$ are hrs.

$f(0)$ is concave down. $f(x)$ concave up.

$f''(x)$ dec. $f'(x)$ inc.

1pm

Since $f'(x) > 1$ °F/hr. all day long.

Meaning $f''(x)$ was pos. all day long.

Says $f'(x)$ is increasing all day long.

$f(x)$

C.A. C.U. C.A. C.U.

Inc. Inc.

1pm

Sketch of $f(x)$
10. Sketch a graph of a function that meets these conditions.

\( f(x) \) is positive for \( x < 0 \)
\( f'(x) > 0 \) for \( x < 3 \)
\( f'(x) < 0 \) for \( x > 3 \)
\( f''(x) < 0 \) for \( x > 0 \)
\( f''(x) \geq 0 \) for \( x < 0 \)
\( f'(3) = 0 \)

correction.

above \( x \)-axis
for \( x < 0 \)
11. Here is the graph of $f(x)$. 

(a) On what intervals is $f(x)$ increasing?  
$f'(x) \text{ pos.}$  
$x < 2 \quad 4 < x < 8$  
$x > 9$ 

(b) On what intervals is $f(x)$ decreasing?  
$f'(x) \text{ neg.}$  
$0 < x < 4 \quad x > 9$ 

(c) On what intervals is $f(x)$ concave up?  
$f''(x) \text{ inc.}$  
$x < 2 \quad 6 < x < 9$ 

(d) On what intervals is $f(x)$ concave down?  
$f''(x) \text{ dec.}$  
$2 < x < 6 \quad x > 9$ 

(e) Use the above information to sketch a graph of $f(x)$.

---

Title: Feb 26-2:15 PM (12 of 12)