Week in Review # 5  
Sections 2.2, 2.3, 2.4

Things to know:
- Be able to sketch the graph of a derivative.
- Be able to give units for the derivative and interpret a derivative.
- Be able to use the derivative to estimate values of a function.
- Know the relationships between the function, first derivative, and the second derivative.
- Be able to sketch a graph of a function given information about its first and second derivatives.

1. Fill in the blanks with the relationships between \( f(x) \), \( f'(x) \), and \( f''(x) \).

- \( f'(x) > 0 \) means that ________________
- \( f'(x) < 0 \) means that ________________
- \( f''(x) > 0 \) means that ________________ and ________________
- \( f''(x) < 0 \) means that ________________ and ________________

2. Sketch the graphs of the derivatives of each of these functions.

(a) \[ f(x) \]

(b) \[ g(x) \]

(c) \[ h(x) \]

(d) \[ k(x) \]
3. Here is the graph of the function $f(x)$.

(a) Arrange the derivatives at the given points from smallest to largest.

(b) At which points does $f'(x)$ and $f''(x)$ have the same sign?

4. Match the points with the derivatives.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f'(x)$</th>
<th>$f''(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>$-2$</td>
<td>$-2$</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>$-4$</td>
</tr>
</tbody>
</table>

5. Suppose $H = f(t)$ is the time, in minutes, that it takes a deep fryer to heat up to $t^\circ\text{F}$.

(a) What are the units of $f'(t)$ and what is the sign of $f'(t)$?

(b) What is the meaning of $f(350) = 15$?

(c) what is the meaning of $f'(350) = 0.25$?

(d) Estimate the time for the deep fryer to heat up to $375^\circ\text{F}$.
6. Suppose $P(t)$ is the monthly payment, in dollars, on a mortgage which will take $t$ years to pay off.

(a) What are the units of $P'(t)$ and the sign of $P'(t)$?

(b) What is the practical meaning of $P'(t)$?

7. Suppose $g(20) = 125$ and $g'(20) = -8$. Estimate $g(18)$, $g(25)$, and $g(31)$.

8. If $f(3) = 20$, $f'(3) = 2$ and $f''(x) < 0$ for $x \geq 3$, what can you say about the value of $f(7)$?
9. The temperature inside a house was given by \( f(t) \) in °F. At 1pm, the temperature was 70°F. The first derivative, \( f'(t) \) decreased until reaching a value of 1°F/hour at 1pm, then increased for the rest of the day. sketch a graph of the temperature inside the house during this time period.

10. Sketch a graph of a function that meets these conditions.
   \( f(x) \) is positive for \( x < 0 \)
   \( f'(x) > 0 \) for \( x < 3 \)
   \( f'(x) < 0 \) for \( x > 3 \)
   \( f''(x) < 0 \) for \( x > 0 \)
   \( f''(x) > 0 \) for \( x < 0 \)
   \( f'(3) = 0 \)
11. Here is the graph of $f'(x)$.

(a) On what intervals is $f(x)$ increasing?

(b) On what intervals is $f(x)$ decreasing?

(c) On what intervals is $f(x)$ concave up?

(d) On what intervals is $f(x)$ concave down?

(e) Use the above information to sketch a graph of $f(x)$. 