

Week in Review # 6
 Sections 3.1, 3.2, 3.3, 3.4, 3.5

1. Find the derivatives for the following functions. Assume that a, b, and k are constants.

$$(a) y = \frac{3}{t^2} - 4t^6 + 5 = 3t^{-2} - 4t^6 + 5$$

$$\frac{dy}{dt} = y' = -6t^{-3} - 24t^5 = -\frac{6}{t^3} - 24t^5$$

$$(b) u = 3aq^4 + q^{1.45} + b^3$$

$$u' = 12aq^3 + 1.45q^{.45}$$

$$(c) f(x) = 6\sqrt[3]{x^4} + \frac{k}{2x^5} = 6x^{\frac{4}{3}} + \frac{k}{2}x^{-5}$$

$$f'(x) = 6 \cdot \frac{4}{3}x^{\frac{1}{3}} + \frac{-5k}{2}x^{-6} = 8x^{\frac{1}{3}} - \frac{5k}{2}x^{-6}$$

$$(d) y = \frac{5x^4 + 7x^2 - 2x + 3}{x}$$

quotient rule $\rightarrow y' = \frac{x(20x^3 + 14x - 2) - (5x^4 + 7x^2 - 2x + 3)(1)}{x^2}$

$$y = \frac{5x^4}{x} + \frac{7x^2}{x} - \frac{2x}{x} + \frac{3}{x} = 5x^3 + 7x - 2 + 3x^{-1}$$

$$y' = 15x^2 + 7 - 3x^{-2}$$

2. Find the equation of the tangent line for the function $y = x^3 e^{(x^2-4)} + 3x$ at $x = 2$

need point & slope.

point: $x=2$ means $y = 2^3 e^{(2^2-4)} + 3(2)$
 $y = 8e^0 + 6$
 $y = 14$

slope: $m_{\text{tan}} = y'(2)$

$$y' = \underbrace{3x^2}_{f'} \underbrace{e^{x^2-4}}_g + \underbrace{x^3}_f \cdot \underbrace{2x e^{x^2-4}}_{g'} + 3$$

$$m_{\text{tan}} = y'(2) = \left. \frac{dy}{dx} \right|_{x=2} =$$

$$\begin{aligned} y'(2) &= 3(2)^2 e^{2^2-4} + 2^3 \cdot 2(2) e^{2^2-4} + 3 \\ &= 12e^0 + 32e^0 + 3 \\ &= 47 \end{aligned}$$

Answer

$$y - 14 = 47(x - 2)$$

use product Rule.
 $y = f \cdot g$
 $y' = f'g + fg'$

$$\begin{aligned} y &= e^{f(x)} \\ y' &= f'(x) e^{f(x)} \end{aligned}$$

3. Find the value for a for the function $y = 4x^3 + ax^2 + 3x + 2$ so that the rate of change
at $x = 1$ is 10.

$$y' = 12x^2 + 2ax + 3$$

↓

$$y'(1) = 10$$

$$10 = 12 + 2a + 3$$

$$10 = 15 + 2a$$

$$-5 = 2a$$

$$\frac{-5}{2} = a$$

4. Find the values of x where the function $y = x^4 - 4x^2 + 10x + 5$ has an instantaneous rate of change of 10.

$$y' = 4x^3 - 8x + 10$$

$$10 = 4x^3 - 8x + 10$$

$$0 = 4x^3 - 8x$$

$$0 = 4x(x^2 - 2)$$

$$4x = 0 \quad \text{or} \quad x^2 - 2 = 0$$

$$x = 0$$

$$x^2 = 2$$

$$x = \pm \sqrt{2}$$

Answer $x = 0, -\sqrt{2}, \sqrt{2}$

5. Suppose the depth of the water, in meters, is a function of time, in hours, since 6am is given by $y = 7 + 3.8 \sin(0.628x)$. How quickly is the water rising or falling at 9am? at Noon?

$$y' = 3.8(.628) \cos(.628x)$$

$$y' = 2.3864 \cos(.628x)$$

at 9AM $y'(3) = -0.73526 \text{ m/hr}$

at noon $y'(6) = -1.9333 \text{ m/hr}$

6. If the position function for an object is given by $s(x) = 7x^3 - 15x^2 - x + 25$. Find the velocity function and the acceleration function.

position $s(t)$

velocity $v(t) = s'(t)$

acceleration $a(t) = v'(t) = s''(t)$

$$v(t) = 21x^2 - 30x - 1$$

$$a(t) = 42x - 30$$

7. Find the derivative for the following functions.

$$(a) f(x) = \sqrt[6]{x^3 + 7x^2 + 6} = (x^3 + 7x^2 + 6)^{1/6}$$

$$f'(x) = \frac{1}{6} (x^3 + 7x^2 + 6)^{-5/6} \cdot (3x^2 + 14x)$$

$$(b) h(t) = 3e^{3t^2+4} + \cancel{5^{4t-5}}$$

$$h'(t) = 3 \cdot (6t) e^{3t^2+4} + 4(5^{4t-5}) \cdot \ln(5)$$

$$= 18t e^{3t^2+4} + 4 * 5^{4t-5} * \ln(5)$$

$$y = a^{f(x)}$$

$$y' = f'(x) a^{f(x)} \ln(a)$$

product Rule.

$$(c) y = (x^3 + 5x) \cos(x^4)$$

$$\begin{aligned} y' &= (3x^2 + 5) \cos(x^4) + (x^3 + 5x) \cdot (-4x^3) \sin(x^4) \\ &= (3x^2 + 5) \cos(x^4) - (x^3 + 5x) (4x^3) \sin(x^4) \end{aligned}$$

$$(d) y = 2^{\sin(3x)} + \ln(x^4 + 7x^3 + 15)$$

$$y' = 3 \cos(3x) \cdot 2^{\sin(3x)} \cdot \ln(2) + \frac{4x^3 + 21x^2}{x^4 + 7x^3 + 15}$$

$$(e) g(x) = e^{-5x^2} \sqrt{x^8 + x - 6} = e^{-5x^2} (x^8 + x - 6)^{1/2}$$

$$g'(x) = -10x e^{-5x^2} (x^8 + x - 6)^{1/2} + e^{-5x^2} \cdot \frac{1}{2} (x^8 + x - 6)^{-1/2} \cdot (8x^7 + 1)$$

$$(f) y = \frac{x^2 + 5}{x^4 - 2x - 1}$$

$$y = \frac{H}{L}$$

$$y' = \frac{L \, dH - H \, dL}{L^2}$$

$$y' = \frac{(x^4 - 2x - 1) \cdot 2x - (x^2 + 5)(4x^3 - 2)}{(x^4 - 2x - 1)^2}$$

$$(g) \ y = \left(\frac{x^4 + 7x + 1}{\cos(5x)} \right)^4$$

$$y' = 4 \left(\frac{x^4 + 7x + 1}{\cos(5x)} \right)^3 \cdot \left[\frac{\cos(5x) \cdot (4x^3 + 7) - (x^4 + 7x + 1) \cdot (-5 \sin(5x))}{(\cos(5x))^2} \right]$$

$$(h) \ g(a) = \ln(2a + e^{\sin(3a)})$$

$$g'(a) = \frac{2 + 3 \cdot \cos(3a) e^{\sin(3a)}}{2a + e^{\sin(3a)}}$$

$$(i) y = \ln\left(\frac{7x^2 + 5}{10 - x^5}\right) = \ln(7x^2 + 5) - \ln(10 - x^5)$$

$$y' = \frac{14x}{7x^2 + 5} - \frac{-5x^4}{10 - x^5}$$

$$y' = \frac{14x}{7x^2 + 5} + \frac{5x^4}{10 - x^5}$$

$$(j) f(x) = \ln((x^4 + 5)^7 \cos(3x^2))$$

$$f(x) = \ln(x^4 + 5)^7 + \ln(\cos(3x^2))$$

$$= 7 \ln(x^4 + 5) + \ln(\cos(3x^2))$$

$$f'(x) = 7 \cdot \frac{4x^3}{x^4 + 5} + \frac{-6x \sin(3x^2)}{\cos(3x^2)}$$

f
g

(k) $y = 7^{(2-3x^2)} \ln(5-2x)$

$$y' = \underbrace{-6x \cdot 7^{2-3x^2} \cdot \ln(7)}_{f'} \cdot \underbrace{\ln(5-2x)}_g + \underbrace{7^{2-3x^2}}_f \cdot \underbrace{\frac{-2}{5-2x}}_{g'}$$

$$y = a^{f(x)}$$

$$y' = f'(x) a^{f(x)} \cdot \ln(a)$$