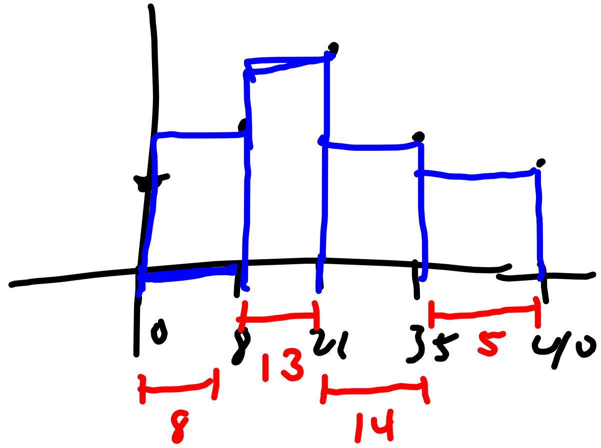
Week in Review # 8
Sections 5.2, 5.3, 5.4, 5.5

1. Use the table to estimate $\int_0^{40} f(x)dx$ with a right sum.

| x | 0 | 8 | 21 | 35 | 40 |
|---|---|---|---|---|---|
| f(x) | 370 | 425 | 440 | 407 | 393 |




base (height)

$$8(425) + 13(440) + 14(407) + 5(393) =$$

$$\int_0^{40} f(x)dx \approx 16783$$

2. Use the figure and a left sum with $n = 4$ to estimate $\int_2^{10} f(x)\,dx$

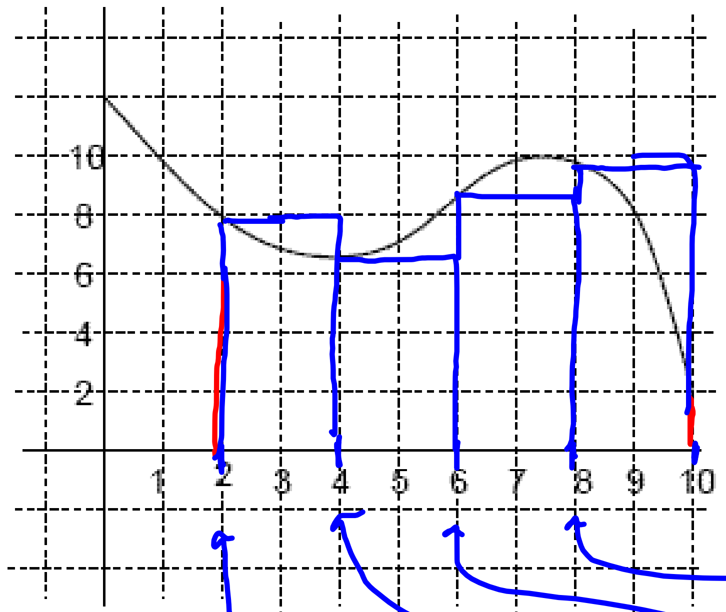
$a = 2$

$b = 10$

$\Delta x = \text{base} = \frac{b-a}{n}$

$= \frac{10-2}{4}$

$= \frac{8}{4} = 2$




$2(f(2)) + 2\cdot f(4) + 2\cdot f(6) + 2\cdot f(8)$

$2(8) + 2(6.5) + 2\cdot(8.5) + 2\cdot(9.5) = 65$

$\int_2^{10} f(x)\,dx \approx 65$

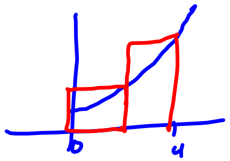
3. Estimate the value of $\int_0^4 e^{x^2}\,dx$ using a right sum with $n = 2$. Is this a lower or upper estimate?

$f(x) = e^{x^2}$ $\quad \hookrightarrow \text{base} = \frac{4-0}{2} = 2$

| x | 0 | 2 | 4 |
|---|---|---|---|
| f(x) | 1 | 54.598 | 8886110.52 |

$$2(54.598) + 2(8886110.52) = \boxed{17772230.24}$$

graph of $e^{x^2}$ on $[0,4]$

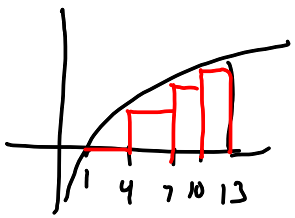



since Rect. all go above this is an over (upper) estimate.

4. Estimate the value of $\int$ [illegible]$)\,dx$ using a left sum with $n = 4$. Is this a lower or upper estimate?

correction $= \int_1^{13} \ln(x)\,dx$

$\text{base} = \frac{13-1}{4} = \frac{12}{4} = 3$




Left. Rect. are below the graph so estimate is a lower estimate.

| x | 1 | 4 | 7 | 10 | 13 |
|---|---|---|---|---|---|
| ln(x) | 0 | 1.3863 | 1.9459 | 2.3026 | 2.5649 |

$$3(0) + 3(1.3863) + 3(1.9459) + 3(2.3026)$$
$$= 16.9044$$

5. Compute the values of these definite integrals.

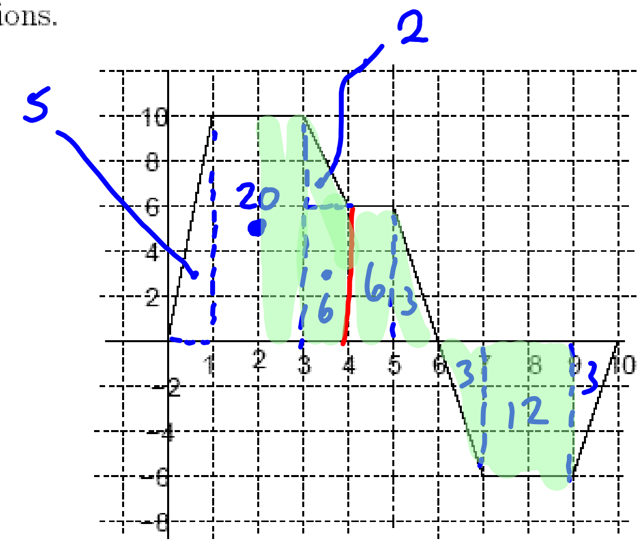
(a) $\int_{1}^{13} \ln(x)\, dx$ = 21.3443

fnInt( ln(x), x, 1, 13) =

math choice #9

(b) $\int_{3}^{7} \frac{2x^4}{x^2+8}\, dx$ = 163.49997

6. Use the graph to answer these questions.




(a) $\int_0^4 f(x)\,dx =$ $5+20+2+6$

$= 33$

(b) $\int_6^{10} f(x)\,dx =$ $-18$

(c) $\int_4^4 f(x)\,dx =$ $0$

since start & stop at the same place

(d) $\int_2^9 f(x)\,dx =$ $27 - 15 = 12$

above: $\left.\begin{matrix}10\\6\\6\\2\\3\end{matrix}\right\} 27$

below $3+12=15$

(e) Find the area between the function and the x-axis from $x=0$ to $x=10$

from 0 to 6 $\rightarrow$ $5+20+2+6+6+3 = 42$

6 to 10 $\rightarrow$ $3+12+3 = 18$

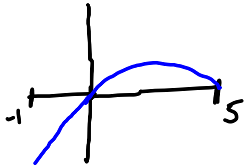
Area $= 42+18 = 60$

7. Does $\int_{-1}^{5}(5x - x^2)\,dx$ represent an area or a difference of areas? Justify your answer.

$\int_{-1}^{5} 5x - x^2\,dx = 18$

This doesn't tell anything

look at the graph

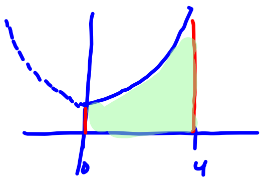



includes Area above + below X-axis so the integral is a difference of Areas.

8. Find the area between the given function and the x-axis from $x = 0$ to $x = 4$.

Look at The graph

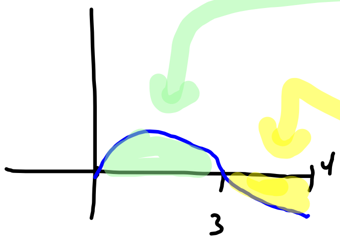
(a) $y = e^{x^2}$




$\int_0^4 e^{x^2}\,dx = 1,149,400.635$

make definite integral pos.

(b) $y = 3x - x^2$




$\boxed{\int_0^3 3x - x^2\,dx} + \left|\int_3^4 3x - x^2\,dx\right|$

$4.5 + |-1.8333|$

$= 6.3333$

or $6\frac{1}{3}$

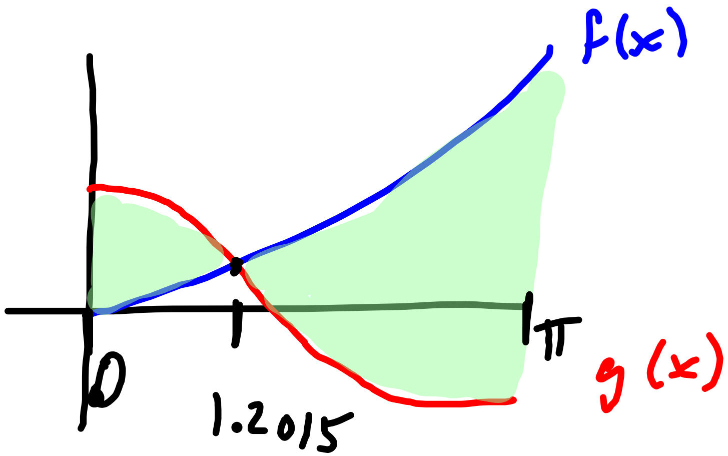
9. Given the functions $f(x) = x^2$ and $g(x) = 4\cos(x)$

(a) Set up the integral(s) that represent the ~~are~~ area between these functions from $x = 0$ to $x = \pi$

(b) Compute the area represented in part (a).

formula

$$\int_A^B \text{top} - \text{bottom} \; dx$$

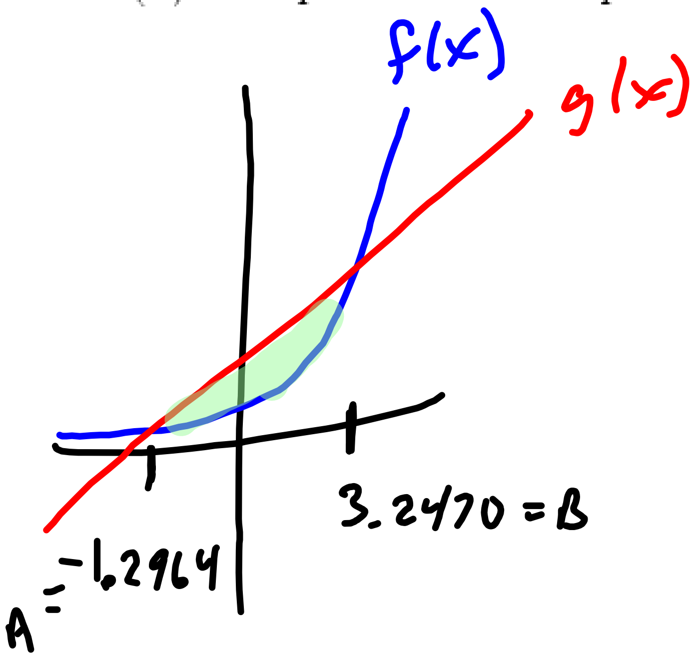



$$\int_0^{1.2015} g(x) - f(x)\,dx + \int_{1.2015}^{\pi} f(x) - g(x)\,dx = 16.6398$$

10. Given the functions $f(x) = 2^x$ and $g(x) = 2x + 3$

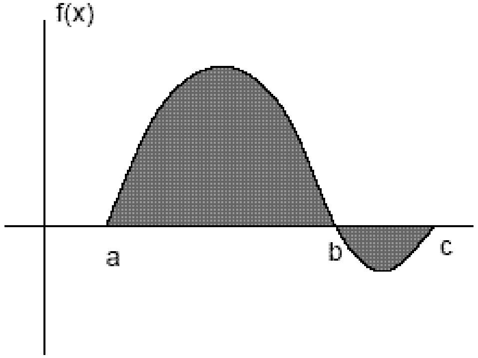
(a) Set up the integral(s) that represent the are bounded between these functions.

(b) Compute the area represented in part (a).




$$\int_A^B g(x) - f(x)\, dx = 9.3831$$

11. use the graph below to arrange the definite integrals from smallest to largest




$A = \int_a^b f(x)\,dx$ $\quad B = \int_b^c f(x)\,dx$ $\quad C = \int_a^c f(x)\,dx$ $\quad D = \int_a^c |f(x)|\,dx$

$+ \qquad - \qquad + \qquad +$

Smallest Largest.

B, C, A, D

12. If $k(t)$ is measured in km/hr$^2$ and $t$ is measured in hours, what are the units of $\int_a^b k(t)\,dt$?

$$\frac{Km}{hr^2} \cdot hr = Km/hr$$

13. If $m'(x) = 0.8x^2 - 6x + 7$ is the rate a certain population of game fish is changing over the years, where x is measured in the number of years from 1995 and $m'(x)$ is measured in millions of fish/yr. Evaluate the definite integral and interpret

(a) $\int_2^8 m'(x)\,dx.$ = -3.6 millions of fish

$\frac{\text{millions of fish}}{\cancel{yr}} \cdot \cancel{yr}$

from 1997 to 2003, the population of this game fish decreased by 3.6 million.

(b) $\int_5^8 m'(x)\,dx.$ = 7.2 millions of fish.

from 2000 to 2003 the population of this game fish had an increase of 7.2 million

(c) The population of this fish was estimated to be 40 million in the year 1998. Estimate the population in 2005.

↳ 10 ↳ 3

$$\int_3^{10} m'(x)\,dx = \underbrace{m(10)}_{\text{pop. in 2005}} - \underbrace{m(3)}_{\text{pop. in 1998}}$$

$$m(3) + \int_3^{10} m'(x)\,dx = m(10)$$

$$40 + 35.4667 = \boxed{75.4667 \text{ million}}$$

population in 2005

14. When a cup of hot chocolate is bought its temperature is 175$^o$F. The outside temperature is 50$^o$F and the rate at which the chocolate is cooling is given by $T'(x) = -10e^{-0.08x}$, where x is measured in minutes and $T'(x)$ is measured in degrees per minute. What is the temperature of the chocolate after 10 minutes? after 1 hour?

10 min

$$175 + \int_0^{10} T'(x)\,dx = 106.1661^\circ F$$

1 hr

$$175 + \int_0^{60} T'(x)\,dx = 51.0287^\circ F$$