Week in Review # 8
Sections 5.2, 5.3, 5.4, 5.5

Things to know:
- Understand that $\Delta t = \frac{b-a}{n}$ is the base of the rectangles.
- Be able to estimate the definite integral (total change) using a graph or by rectangles.
- Be able to find the area between a function and the x-axis on an interval.
- Be able to find the area bounded between two functions.
- Know how to find the units of an integral and interpret the meaning of an integral.

1. Use the table to estimate $\int_{0}^{40} f(x)dx$ with a right sum.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>8</th>
<th>21</th>
<th>35</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>370</td>
<td>425</td>
<td>440</td>
<td>407</td>
<td>393</td>
</tr>
</tbody>
</table>

2. Use the figure and a left sum with $n = 4$ to estimate $\int_{2}^{10} f(x) dx$.

3. Estimate the value of $\int_{0}^{4} e^{x^{2}} dx$ using a right sum with $n = 2$. Is this a lower or upper estimate?
4. Estimate the value of $\int_{1}^{13} \ln(x) \, dx$ using a left sum with $n = 4$. Is this a lower or upper estimate?

5. Compute the values of these definite integrals.
   (a) $\int_{1}^{13} \ln(x) \, dx$
   (b) $\int_{3}^{7} \frac{2x^4}{x^4 + 8} \, dx$

6. Use the graph to answer these questions.
   (a) $\int_{0}^{4} f(x) \, dx =$
   (b) $\int_{6}^{10} f(x) \, dx =$
   (c) $\int_{4}^{4} f(x) \, dx =$
   (d) $\int_{2}^{9} f(x) \, dx =$
   (e) Find the area between the function and the x-axis from $x = 0$ to $x = 10$
7. Does \( \int_{-1}^{5} (5x - x^2) \, dx \) represent an area or a difference of areas? Justify your answer.

8. Find the area between the given function and the x-axis from \( x = 0 \) to \( x = 4 \).
   
   (a) \( y = e^{x^2} \)

   (b) \( y = 3x - x^2 \)

9. Given the functions \( f(x) = x^2 \) and \( g(x) = 4 \cos(x) \)
   
   (a) Set up the integral(s) that represent the area between these functions from \( x = 0 \) to \( x = \pi \)

   (b) Compute the area represented in part (a).
10. Given the functions \( f(x) = 2^x \) and \( g(x) = 2x + 3 \)

(a) Set up the integral(s) that represent the area bounded between these functions.

(b) Compute the area represented in part (a).

11. Use the graph below to arrange the definite integrals from smallest to largest

\[
A = \int_{a}^{b} f(x) \, dx \quad B = \int_{b}^{c} f(x) \, dx \quad C = \int_{a}^{c} f(x) \, dx \quad D = \int_{a}^{c} |f(x)| \, dx
\]

12. If \( k(t) \) is measured in km/hr\(^2\) and \( t \) is measured in hours, what are the units of \( \int_{a}^{b} k(t) \, dt \)?
13. If \( m'(x) = 0.8x^2 - 6x + 7 \) is the rate a certain population of game fish is changing over the years, where \( x \) is measured in the number of years from 1995 and \( m'(x) \) is measured in millions of fish per year. Evaluate the definite integral and interpret

(a) \( \int_{2}^{8} m'(x) \, dx \).

(b) \( \int_{5}^{8} m'(x) \, dx \).

(c) The population of this fish was estimated to be 40 million in the year 1998. Estimate the population in 2005.

14. When a cup of hot chocolate is bought its temperature is 175\(^\circ\)F. The outside temperature is 50\(^\circ\)F and the rate at which the chocolate is cooling is given by \( T'(x) = -10e^{-0.08x} \), where \( x \) is measured in minutes and \( T'(x) \) is measured in degrees per minute. What is the temperature of the chocolate after 10 minutes? after 1 hour?