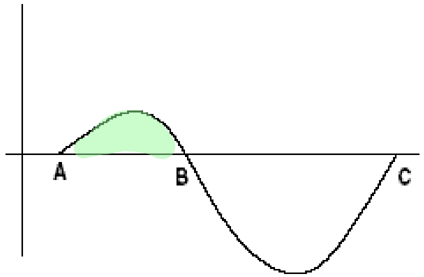
Week in Review # 9
Sections 5.5, 7.1, 7.2

1. Use the graph is of $f'(x)$ to determin which of these two values are the largest.



$\int_J^K f'(x)\,dx = f(K) - f(J)$

(a) $f(a)$ or $f(b)$

$\boxed{\int_a^b f'(x)} = f(b) - f(a)$

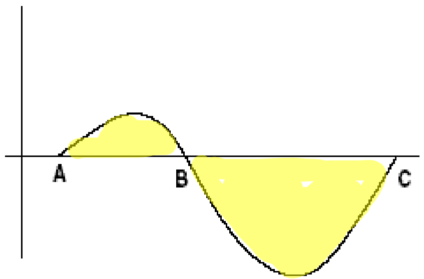
positive

Bigger since

$f(b) - f(a) > 0$

implies $f(b) > f(a)$

(b) $f(a)$ or $f(c)$



$\boxed{\int_a^c f'(x)\,dx} = f(c) - f(a)$

Neg. result

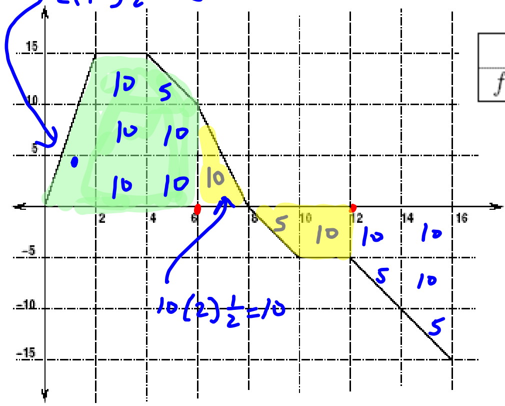
Since more Area below

$f(c) - f(a) < 0$

$f(c) < f(a)$

2. The graph is of $f'(x)$. Fill in the table values for $f(x)$ given that $f(6) = 100$

$2(15)\frac{1}{2} = 15$



$10(2)\frac{1}{2} = 10$

x	0	2	4	12	14	16
$f(x)$	30	45	75	95	80	55

$\int_0^6 f'(x)\,dx = f(6) - f(0)$

evaluate $\int$ with the graph.

$70 = f(6) - f(0)$

$70 = 100 - f(0)$

$-30 = -f(0)$

$30 = f(0)$

$\int_0^2 f'(x)\,dx = f(2) - f(0)$

$15 = f(2) - 30$

$45 = f(2)$

$\int_2^4 f'(x)\,dx = f(4) - f(2)$

$30 = f(4) - 45$

$75 = f(4)$

$\int_6^{12} f'(x)\,dx = f(12) - f(6)$

$-5 = f(12) - 100$

$95 = f(12)$

$\int_{12}^{14} f'(x)\,dx = f(14) - f(12)$

$-15 = f(14) - 95$

$80 = f(14$

$\int_{14}^{16} f'(x)\,dx = f(16) - f(14)$

$-25 = f(16) - 80$

$55 = f(16)$

3. Which of these is an antiderivative of $f'(x) =$ [redacted] $4xe^{2x}$?

- ~~(a)~~ $f(x) = 2x^2e^{2x} + xe^{2x} + C$
- (b) $f(x) = 2xe^{2x} - e^{2x} + C$ ✓
- (c) $f(x) = (2x-1)e^{2x}$ ✓

  $= 2xe^{2x} - e^{2x} + 0$

take deriv. + see which gets result of $4xe^{2x}$.

a) $f'(x) = 4xe^{2x} + 2x^2 \cdot 2 \cdot e^{2x} + 1e^{2x} + x \cdot 2 \cdot e^{2x} + 0$

$= 4xe^{2x} + 4x^2e^{2x} + e^{2x} + 2xe^{2x}$

$= 6xe^{2x} + 4x^2e^{2x} + e^{2x}$

will not simplify to $4xe^{2x}$ so not an anti deriv.

b) $2e^{2x} + 2x \cdot 2e^{2x} - 2e^{2x} + 0$

$= 4xe^{2x}$

yep.

c) works since we are taking the deriv. to get $4xe^{2x}$.

4. Find $f(x)$ if $f'(x) = 6e^{2x} + 8\sin(2x)$ and $f(0) = 20$

$\int e^{2x}\,dx = \frac{1}{2}e^{2x} + c$

$\int \sin(2x)\,dx = -\frac{1}{2}\cos(2x) + c$

$$f(x) = \frac{6}{2}e^{2x} + -\frac{8}{2}\cos(2x) + C$$

$$= 3e^{2x} - 4\cos(2x) + C$$

$$20 = f(0) = 3e^{0} - 4\cos(0) + C$$

$$20 = 3 - 4 + C$$

$$20 = -1 + C$$

$$21 = C$$

$$\boxed{f(x) = 3e^{2x} - 4\cos(2x) + 21}$$

5. Compute these integrals.

(a) $\int 7x^4 + 5x^3 + 8\,dx$ $= \dfrac{7x^5}{5} + \dfrac{5x^4}{4} + 8x + C$

(b) $\int 3x^2\,dw$ $= 3x^2 w + C$

variable is w

(c) $\int 5x + \frac{5}{x} + \frac{6}{x^4}\,dx$ $= \int 5x + 5x^{-1} + 6x^{-4}\,dx$

$= \dfrac{5x^2}{2} + 5\ln|x| + \dfrac{6x^{-3}}{-3} + C$

$\int x^{-2}\,dx = \dfrac{x^{-1}}{-1} + C = -\dfrac{1}{x} + C$

(d) $\int \frac{1}{e^{3x}} + \frac{1}{4x}\,dx$ $= \int e^{-3x} + \frac{1}{4}x^{-1}\,dx$

$$= \frac{1}{-3}e^{-3x} + \frac{1}{4}\ln|x| + C$$

(e) $\int x^2\sqrt{x}\,dx$ $= \int x^2 \cdot x^{.5}\,dx = \int x^{2.5}\,dx$

$$= \frac{x^{3.5}}{3.5} + C$$

(f) $\int 6\cos(3x) + e\,dx$ $= \frac{6}{3}\sin(3x) + e \cdot x + C$

just a #.

$$\boxed{\int \cos(kx)\,dx = \frac{1}{k}\sin(kx) + C}$$

(g) $\int 4x^3(1+2x^4)^8\,dx = \int 4u^8 x^3dx$

$u = 1+2x^4$

$\frac{du}{dx} = 8x^3$

$\boxed{\frac{du}{8}} = x^3dx$

$= \int 4u^8 \frac{1}{8}du$

$= \int \frac{1}{2}u^8 du$

$= \frac{1}{2}\frac{u^9}{9} + C$

$= \frac{1}{18}(1+2x^4)^9 + C$

$(\text{junk})^{\text{power}}$

e^{junk}

$\frac{\text{something}}{\text{junk}}$

$\sin(\text{junk})$

$\cos(\text{junk})$

Let

$u = \text{junk}$

(h) $\int \frac{x^4}{\sqrt{x^5+5}}\,dx$ $= \int \frac{1}{u^{1/2}} \cdot \frac{du}{5} = \int \frac{1}{5} u^{-1/2}\,du$

$u = x^5 + 5$

$\frac{du}{dx} = 5x^4$

$du = 5x^4\,dx$

$\frac{du}{5} = x^4\,dx$

$= \frac{1}{5} \cdot \frac{u^{1/2}}{1/2} + C$

$= \frac{2}{5} u^{1/2} + C$

$$\boxed{= \frac{2}{5}(x^5+5)^{1/2} + C}$$

(i) $\int (8x^3 + 14)\sin(x^4 + 7x)\, dx = \int 2(4x^3+7)\sin(u)\, dx$

$u = x^4 + 7x$

$\frac{du}{dx} = 4x^3 + 7$

$du = (4x^3+7)\, dx$

$= \int 2\sin(u)\, du$

$= -2\cos(u) + C$

$= -2\cos(x^4+7x) + C$

(j) $\int \sin(2x)\left(\cos(2x) + 10\right)^7 dx$ $= \int -\frac{1}{2} u^7 \, du$

$u = \cos(2x) + 10$ $= -\frac{1}{2} \frac{u^8}{8} + C$

$\frac{du}{dx} = -2\sin(2x)$

$$\boxed{= -\frac{1}{16}\left(\cos(2x)+10\right)^8 + C}$$

$\frac{du}{-2} = \sin(2x)\,dx$

$-\frac{1}{2} du$

(k) $\int \frac{12xe^{x^2}}{1+e^{x^2}}\,dx$ $= \int \frac{12}{u} \frac{du}{2} = \int \frac{6}{u}\,du$

$= \int 6u^{-1}\,du$

$= 6\ln|u| + C$

$= 6\ln\left(1+e^{x^2}\right) + C$

$u = 1+e^{x^2}$

$\frac{du}{dx} = 2xe^{x^2}$

$\frac{du}{2} = xe^{x^2}\,dx$

(1) $\int 12\sin(6x+7)\cos(6x+7)\,dx$ $= \int 12u\,\frac{du}{6} = \int 2u\,du$

$u = \sin(6x+7)$

$= u^2 + C$

$\frac{du}{dx} = 6\cos(6x+7)$

$= (\sin(6x+7))^2 + C$

$\boxed{\frac{du}{6}} = \cos(6x+7)\,dx$

$= \int 12u\,\frac{du}{-6} = \int -2u\,du$

$u = \cos(6x+7)$

$= -u^2 + C$

$\frac{du}{dx} = -6\sin(6x+7)$

$= -(\cos(6x+7))^2 + C$

$\frac{du}{-6} = \sin(6x+7)\,dx$