

Chapter 2 Homework Solutions

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1. x = the number of dimes
 y = the number of quarters

$$x + y = 32$$

$$.1x + .25y = 5.15$$

Solution:

19 dimes
 13 quarters

2. x = the number of nickels
 y = the number of quarters

$$x + y = 150$$

$$.05x + .25y = 18.50$$

Solution:

95 nickels
 55 quarters

3. x = amount invested in Fund A
 y = amount invested in Fund B

$$x + y = 50000$$

$$.074x + .098y = 4072$$

Solution:

\$34,500 invested in Fund A
 \$15,500 invested in Fund B

4. x = number of five dollar bills
 y = number of ten dollar bills
 z = number of twenty dollar bills

$$x + y + z = 70$$

$$5x + 10y + 20z = 740$$

$$x - 3y = 0$$

Solution:

36 five dollar bills
 12 ten dollar bills
 22 twenty dollar bills

5. x = number of minutes jogging
 y = number of minutes playing handball
 z = number of minutes riding a bike

$$x + y + z = 60$$

$$11x + 13y + 5z = 660$$

$$x - 2z = 0$$

Solution:

20 minutes jogging
 30 minutes playing handball
 10 minutes riding a bike

6. x = number of hours Valley Mills is scheduled.
 y = number of hours Marlin is scheduled.
 z = number of hours Hillsboro is scheduled.

$$10x + 7y + 5z = 1365$$

$$12x + 10y + 4z = 1530$$

$$6x + 8y + 13z = 1890$$

Solution:

Valley Mills scheduled for 60 hours
 Marlin scheduled for 45 hours
 Hillsboro scheduled for 90 hours

7. x = the number of carton A
 y = the number of carton B
 z = the number of carton C

$$2x + 6y + 4z = 50$$

$$5x + 8y + 6z = 78$$

$$3x + 2y + 10z = 52$$

Solution:

4 of carton A
 5 of carton B
 3 of carton C

8. x = number of plain hamburgers
 y = number of double cheeseburgers
 z = number of regular cheeseburgers

$$x + y + z = 86$$

$$x + 2y + z = 100$$

$$4y + 2z = 140$$

Solution:

30 plain hamburgers
 14 double cheeseburgers
 42 regular cheeseburgers

9. x = number of one-bedroom units
 y = number of two-bedroom units
 z = number of three-bedroom units

$$x + y + z = 225$$

$$y + z = 2x$$

$$x = 3z$$

Solution:

$x=75$
 $y = 125$
 $z = 25$

10. x = number of children at the show
 y = number of students at the show
 z = number of adults at the show

$$x + y + z = 900$$

$$2x + 3y + 4z = 2800$$

$$2z = x + y$$

Solution:

$x=200$
 $y=400$
 $z=300$

11. x = number of barrels of mix A
 y = number of barrels of mix B
 z = number of barrels of mix C
 w = number of barrels of mix D

$$30x + 30y + 30z + 60w = 900$$

$$50z + 75y + 25z + 25w = 750$$

$$30x + 20y + 20z + 50w = 700$$

Initial matrix

final matrix

$$\left[\begin{array}{cccc|c} 30 & 30 & 30 & 60 & 900 \\ 50 & 75 & 25 & 25 & 750 \\ 30 & 20 & 20 & 50 & 700 \end{array} \right] \xrightarrow{\text{rref}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 10 \\ 0 & 1 & 0 & -1 & -5 \\ 0 & 0 & 1 & 2 & 25 \end{array} \right]$$

Parametric Solution:

$$x = 10 - w$$

$$y = -5 + w$$

$$z = 25 - 2w$$

$$w = \text{any number}$$

Now place restrictions on the parameter. Since we are not told a maximum number of barrels that can be bought assume there is no limit. We know the number of barrels bought has to be non-negative.

$$\begin{array}{llll} x \geq 0 & y \geq 0 & z \geq 0 & w \geq 0 \\ 10 - w \geq 0 & -5 + w \geq 0 & 25 - 2w \geq 0 & \\ 10 \geq w & w \geq 5 & 25 \geq 2w & \\ w \leq 10 & & 12.5 \geq w & \\ & & w \leq 12.5 & \end{array}$$

The restriction on the parameter is that w must be an integer and $5 \leq w \leq 10$ (i.e. $w = 5, 6, 7, 8, 9, 10$)

12. x = number of cars purchased with 6,000 gallon capacity
 y = number of cars purchased with 8,000 gallon capacity
 z = number of cars purchased with 18,000 gallon capacity

$$x + y + z = 24$$

$$6000x + 8000y + 18000z = 250000$$

Initial matrix and final matrix:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 24 \\ 6000 & 8000 & 18000 & 250000 \end{array} \right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & -5 & -29 \\ 0 & 1 & 6 & 53 \end{array} \right]$$

Parametric solution:

$$x = -29 + 5z$$

$$y = 53 - 6z$$

$$z = \text{any number}$$

We can not buy a part of a tank car. So z must be an integer. We also know that all of the variables must be greater than or equal to zero.

$$\begin{array}{lll} x \geq 0 & y \geq 0 & z \geq 0 \\ -29 + 5z \geq 0 & 53 - 6z \geq 0 & \\ 5z \geq 29 & 53 \geq 6z & \\ z \geq 5.8 & 53/6 \geq z & \\ & z \leq \frac{53}{6} \approx 8.8333 & \end{array}$$

In addition we know that the variables can not be any larger than 24

$$\begin{array}{lll} x \leq 24 & y \leq 24 & z \leq 24 \\ -29 + 5z \leq 24 & 53 - 6z \leq 24 & \\ 5z \leq 53 & 29 \leq 6z & \\ z \leq 10.6 & 29/6 \leq z & \\ & z \geq \frac{29}{6} \approx 4.8333 & \end{array}$$

Taken all together, we find that $z = 6, 7, 8$. This problem ends up having only three solutions.

13. x = the number of chihuahuas bought
 y = the number of cats bought
 z = the number of dogs bought

$$x + y + z = 14$$

$$4x + 7y + 16z = 116$$

Solution:

$$x = -6 + 3z$$

$$y = 20 - 4z$$

$$z = \text{any number}$$

We can not buy a part of a pet. So z must be an integer. We also know that all of the variables must be greater than or equal to zero.

$$\begin{array}{lll} x \geq 0 & y \geq 0 & z \geq 0 \\ -6 + 3z \geq 0 & 20 - 4z \geq 0 & \\ 3z \geq 6 & 20 \geq 4z & \\ z \geq 2 & 5 \geq z & \end{array}$$

In addition we know that the variables can not be any larger than 14

$$\begin{array}{lll} x \leq 14 & y \leq 14 & z \leq 14 \\ -6 + 3z \leq 14 & 20 - 4z \leq 14 & \\ 3z \leq 20 & 6 \leq 4z & \\ z \leq \frac{20}{3} \approx 6.6667 & 1.5 \leq z & \end{array}$$

Taken all together, we find that $z = 2, 3, 4, \text{ or } 5$

14. s = the number of small shirts at the end of the day.
 m = the number of medium shirts at the end of the day.
 l = the number of large shirt at the end of the day.

$$s + m + l = 45$$

$$8s + 10m + 13l = 480$$

$$8(2s) + 10(4m) + 13(5l) = 1940$$

Solution:

15 small shirts

10 medium shirts

20 large shirts

15. x = number of tank cars purchased with 7,000 gallon capacity
 y = number of tank cars purchased with 9,000 gallon capacity
 z = number of tank cars purchased with 20,000 gallon capacity

$$x + y + z = 40$$

$$7000x + 9000y + 20000z = 400000$$

Solution:

$$x = -20 + 5.5z$$

$$y = 60 - 6.5z$$

$$z = 4, 6, \text{ or } 8$$

16. x = number of wood pens made
 y = number of silver pens made
 z = number of gold pens made

$$x + .5y + 3z = 12000$$

$$2x + 3y + 2z = 9600$$

Solution:

$$x = 15600 - 4z$$

$$y = -7200 + 2z$$

$$3600 \leq z \leq 3900 \text{ and } z \text{ is an integer}$$

- 17.
- x
- = the number of evil sorcerers slain.

 y = the number of warriors slain. z = the number of orcs slain.

$$x + y + z = 370$$

$$2x + 4y + z = 560$$

$$y = 6x$$

Solution:

10 evil sorcerers

60 warriors

300 orcs

- 18.
- x
- = the amount of money invested in the QX company

 y = the amount of money invested in the RY company z = the amount of money invested in the KZ company

$$x + y + z = 17300$$

$$2z = y$$

$$1.5 \left(\frac{x}{130} \right) + 1 \left(\frac{y}{75} \right) + 2 \left(\frac{z}{90} \right) = 251$$

Solution:

\$6,500 invested in QX

\$7,200 invested in RY

\$3,600 invested in KZ

- 19.
- x
- = number of 12-ounce (small) cups sold

 y = number of 16-ounce (medium) cups sold z = number of 20-ounce (large) cups sold

$$x + y + z = 23$$

$$12x + 16y + 20z = 376$$

$$x + 2y + 3z = 48$$

Solution:

$$x = z - 2$$

$$y = -2z + 25$$

$$z = 2, 3, 4, \dots, 12$$

20. (a) the variables
- x
- ,
- y
- ,
- z
- , and
- w
- are the average number of vehicles on that section of the road.

Note: the number of vehicles entering the intersection must equal the number of vehicles exiting the intersection.

$$x + y = 1400$$

$$y + z = 1200$$

$$z + w = 1100$$

$$x + w = 1300$$

Solution:

$$x = 1300 - w$$

$$y = 100 + w$$

$$z = 1100 - w$$

$$w = \text{any number}$$

- (b)
- $200 \leq w \leq 1000$

- (c) To get this restriction we need
- $y = 150$
- . This means that
- $w = 50$
- . Since this is outside of the restrictions found in part b, the answer is no.

21. (a)
- $x + y = 1300$

$$y + z = 1300$$

$$z + w - m = 800$$

$$v - m = 500$$

$$x + w - v = 300$$

Solution:columns in the matrix are x, y, z, v, m, w

$$x = 800 + m - w$$

$$y = 500 - m + w$$

$$z = 800 + m - w$$

$$v = 500 + m$$

$$m = \text{any number}$$

$$w = \text{any number}$$

- (b) The only restriction that we can place on the parameters
- m
- and
- w
- is that they have to be non-negative, i.e.
- ≥ 0
- . We can not say anything else since the choice of
- m
- will affect the possible choices of
- w
- .

22. (a)
$$\begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -4 \\ 0 & 4 \\ 8 & -2 \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ -1 & -2 \\ -5 & 3 \end{bmatrix}$$

- (b) Not possible, the dimension don't match.

(c)
$$\begin{bmatrix} 7 & -14 & 0 \\ -7 & 21 & 14 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 8 \\ -4 & 4 & -2 \end{bmatrix} = \begin{bmatrix} 9 & -14 & 8 \\ -11 & 25 & 12 \end{bmatrix}$$

- (d) Not possible, the dimensions don't match.

(e)
$$\begin{bmatrix} 3 & -6 & 0 \\ -3 & 9 & 6 \end{bmatrix} - \begin{bmatrix} 2 & -2 & 6 \\ 0 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -4 & -6 \\ -3 & 5 & 4 \end{bmatrix}$$

23. (a)
$$\begin{bmatrix} 5a & -15 & 5 \\ 0 & 5 & 20 \end{bmatrix} + \begin{bmatrix} 6a & -6 & 2 \\ 0 & 2 & 6 \end{bmatrix} = \begin{bmatrix} 11a & -21 & 7 \\ 0 & 7 & 26 \end{bmatrix}$$

- (b) Not possible, the dimensions don't match.

- (c) Not possible, the dimensions don't match.

(d)
$$\begin{bmatrix} 6 & 0 \\ -6 & 9 \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 6+a & b \\ -6+c & 9+d \end{bmatrix}$$

(e)
$$\begin{bmatrix} 2 & 6 & 2j \\ -4 & 0 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 3 & -5 \\ 7 & k & 2 \end{bmatrix} - C^T = \begin{bmatrix} 2 & 9 & 2j-5 \\ 3 & k & 6 \end{bmatrix} - \begin{bmatrix} 2 & 7 & 1 & b \\ -1 & 0 & -3 & \end{bmatrix} = \begin{bmatrix} 0 & 8 & 2j-5-b \\ 4 & k & 9 \end{bmatrix}$$

24. (a)
$$\begin{bmatrix} 6x+2 & 12+2u \\ -3-2z & 14 \end{bmatrix} = \begin{bmatrix} 2y & 5 \\ 7 & y \end{bmatrix}$$

Now solve these equations:

$$6x + 2 = 2y$$

$$12 + 2u = 5$$

$$-3 - 2z = 7$$

$$14 = y$$

Answer:

$$y = 14, z = -5, u = -3.5, \text{ and } x = 26/6$$

$$(b) \begin{bmatrix} 3 & -4x + 5y \\ 6y + 10x & 12 \end{bmatrix} = \begin{bmatrix} 3 & 22 \\ -3 & 12 \end{bmatrix}$$

Now solve these equations:

$$-4x + 5y = 22$$

$$6y + 10x = -3$$

$$\text{Answer: } x = \frac{-147}{74} \text{ and } y = \frac{104}{37}$$

$$(c) \begin{bmatrix} 2 & 3x \\ 6x & 6 \end{bmatrix} + 2 \begin{bmatrix} 3 & -2y \\ 6y & -1 \end{bmatrix} = \begin{bmatrix} 8 & -13 \\ 84 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 3x - 4y \\ 6x + 12y & 4 \end{bmatrix} = \begin{bmatrix} 8 & -13 \\ 84 & 4 \end{bmatrix}$$

Now solve these equations:

$$3x - 4y = -13$$

$$6x + 12y = 84$$

Answer: $x = 3$ and $y = 5.5$

$$(d) \begin{bmatrix} x - 6y & 2 - 2z \\ y - 14 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 2x & -1 \end{bmatrix}$$

Now solve these equations:

$$x - 6y = 4$$

$$2 - 2z = 0$$

$$y - 14 = 2x$$

Answer: $x = -8$, $y = -2$, and $z = 1$

25. (a) 3×1
 (b) 5×3
 (c) Not possible.
 (d) 4×4
 (e) Not possible.
 (f) 3×5

26. (a) False. Try with the following.

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

- (b) True
 (c) False, it is a 4×4 matrix.

27. (a) Not possible

$$(b) \begin{bmatrix} 13 & -7 \\ 4 & 3 \end{bmatrix}$$

- (c) Not possible

$$(d) \begin{bmatrix} 2 & -2 & 4 \\ -2 & 7 & 6 \end{bmatrix}$$

$$(e) \begin{bmatrix} 17 & 20 \\ 7 & 6 \end{bmatrix}$$

$$(f) \begin{bmatrix} a - 2c & b - 2d \\ 2c & 2d \\ 4a - c & 4b - d \end{bmatrix}$$

$$(g) \begin{bmatrix} a & -a + 2b & 3a + b \\ c & -c + 2d & 3c + d \end{bmatrix}$$

$$28. \begin{bmatrix} 2 & 6 & -4 \\ 10 & 16 & -8 \\ -12 & 20 & 10 \end{bmatrix}$$

$$29. AB = \begin{bmatrix} x + 5 & 2 \\ y + 5 & 2 \end{bmatrix} \text{ and } BA = \begin{bmatrix} x & 1 \\ 5x + 2y & 7 \end{bmatrix}$$

30. Note: either multiple the entire matrix or only use the row and column needed for the answer.

$$(a) C_{1,3} = 0 + 9 + 20 = 29$$

$$(b) D_{3,1} = 0 + 0 + 50 + 4 = 54$$

$$31. \begin{bmatrix} -5 & 30 \\ 6 + x - 2y & 15 + 5y \end{bmatrix} = \begin{bmatrix} -5 & y + 2z \\ 1 & 35 \end{bmatrix}$$

Now solve these equations:

$$30 = y + 2z$$

$$6 + x - 2y = 1$$

$$15 + 5y = 35$$

Answer: $x = 3$, $y = 4$, and $z = 13$

32. (a) $BM = [2910 \quad 8970]$

There is no meaning for these numbers since the labels of the rows/columns do not match up.

$$(b) ML^T = \begin{bmatrix} 7200 \\ 2700 \end{bmatrix}$$

The 7200 is the amount of vitamin A and the 2700 is the amount of vitamin C that is consumed at lunch.

- (c) $(B + L)M = [6840 \quad 21480]$

There is no meaning for these numbers since the labels of the rows/columns do not match up.

$$(d) M(BL)^T = \begin{bmatrix} 11200 \\ 4040 \end{bmatrix}$$

The 11200 is the amount of vitamin A and the 4040 is the amount of vitamin C that is consumed together at breakfast and lunch.

$$33. (a) \begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 10 \end{bmatrix}$$

also acceptable is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 6 \\ 7 \\ 10 \end{bmatrix}$$

$$(b) \begin{bmatrix} 1 & 0 & 8 \\ 1 & -1 & 2 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 15 \\ 2 \end{bmatrix}$$

also acceptable is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 8 \\ 1 & -1 & 2 \\ 3 & 2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 4 \\ 15 \\ 2 \end{bmatrix}$$

$$34. (a) \begin{bmatrix} -35/32 & 25/32 & -33/32 \\ 3/8 & -1/8 & 1/8 \\ 7/32 & -5/32 & 13/32 \end{bmatrix}$$

(b) not possible.

35. (a) Solve the equation for M.

$$AM - 3B = C$$

$$AM = C + 3B$$

$$M = A^{-1}(C + 3B)$$

$$M = \begin{bmatrix} 11.5 & 25 & 26.1 \\ -2.55 & -10.4 & -3.11 \\ 5.2 & 3.6 & 9.04 \end{bmatrix}$$

(b) Solve the equation for K.

$$KA + KC = B$$

$$K(A + C) = B$$

$$K = B \cdot (A + C)^{-1}$$

$$K = \begin{bmatrix} \frac{427}{141} & \frac{-643}{141} & \frac{10}{47} \\ \frac{-2507}{1692} & \frac{4217}{1692} & \frac{-181}{564} \\ \frac{-169}{846} & \frac{-11}{846} & \frac{-23}{282} \end{bmatrix}$$

(c) Solve the equation for J.

$$3J + CJ = 3A^T$$

$$(3I + C)J = 3A^T$$

$$J = (3I + C)^{-1} \cdot 3A^T$$

$$J = \begin{bmatrix} \frac{-3741}{7937} & \frac{6639}{7937} & \frac{-26169}{7937} \\ \frac{795}{7937} & \frac{174}{7937} & \frac{14001}{7937} \\ \frac{1581}{7937} & \frac{975}{7937} & \frac{2505}{7937} \end{bmatrix}$$

$$36. (a) \begin{bmatrix} 3 & 2 & 1 \\ -3 & 3 & 4 \\ 2 & 2 & 1 \end{bmatrix}$$

$$(b) \begin{bmatrix} 1 & 0 & -1 \\ -2.2 & -2 & 3 \\ 2.4 & .4 & -3 \end{bmatrix}$$

$$37. A^{-1} = \begin{bmatrix} 0.5 & -3 & -4 \\ -0.5 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$$

38. (a) $x = 3 - 4y + w$

$$z = 4 - 2w$$

 $y, w = \text{any number}$ (b) $x = 8, y = 2, \text{ and } z = 5$

(c) No solution.

Note: no solution mean no solution for ALL of the variables. Do not say that $x=8, z=5$, no solution.

(d) $x = 3 + z$

$$y = 1 - 2z$$

 $z = \text{any number}$ (e) $x = 0, y = 4, \text{ and } z = 2$

(f) No solution

(g) $x = -2, y = 1, z = 3$

(h) No solution

$$39. (a) \left[\begin{array}{ccc|c} 1 & 0 & 1 & -12 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & 7 & -7 \end{array} \right]$$

$$(b) \left[\begin{array}{ccc|c} 1 & 0 & 9 & 12 \\ 0 & 2 & 19 & 27 \\ 0 & 2 & -12 & -4 \end{array} \right]$$

$$(c) \left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 0 & -2 & 4 & 4 \\ 0 & 18 & 3 & 10 \\ 3 & 0 & 6 & 1 \end{array} \right]$$

$$(d) \left[\begin{array}{ccc|c} 4 & 0 & 26 & -7 \\ 0 & 4 & -2 & 5 \\ 0 & 7 & 74 & 35 \\ 5 & 1 & 2 & 5 \end{array} \right]$$

40. (a) i. $x = 2 - 2z$

$$y = 2 + z$$

 $z = \text{any number}$

ii. no solution

(b) i. $x = 12, y = -22, \text{ and } z = 41$

$$\text{ii. } x = \frac{-2}{17}, y = \frac{-10}{17}, \text{ and } z = \frac{-60}{17}$$

(c) no solution

(d) $x = 7 - 10z$

$$y = 1 + 3z$$

 $z = \text{any number}$ (e) $x = 8, y = 3, \text{ and } z = 5$ (f) $x = \frac{70}{3} + z$

$$y = \frac{110}{3} - 2z$$

 $z = \text{any number}$ (g) $x = 1, y = -2, z = 3, \text{ and } w = 8$ (h) $x = 26 - 2y + 14w$

$$z = 7 + 6w$$

 $y = \text{any number}$ $w = \text{any number}$