

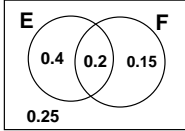
Chapter 7 Homework Solutions

Compiled by Joe Kahlig

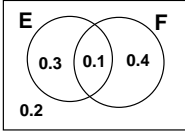
- $S = \{(\text{heads, red}), (\text{heads, white}), (\text{tails, red}), (\text{tails, white})\}$
 - There are multiple answers for this part. Any two disjoint subsets of S are acceptable.
 $E = \{(\text{h,r}), (\text{h,w})\}$
 $F = \{(\text{t,w})\}$
- Since we are drawing them out simultaneously, we don't care about the order. i.e. (1,2) is the same as (2,1)
 $S = \{(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 4), (2, 5), (2, 6), (2, 7), (3, 4), (3, 5), (3, 6), (3, 7), (4, 5), (4, 6), (4, 7), (5, 6), (5, 7), (6, 7)\}$
 - $E = \{(1, 3), (1, 5), (1, 7), (3, 5), (3, 7), (5, 7)\}$
 - $F = \{(2, 4), (2, 6), (4, 6)\}$
 - No.
 - There are multiple answers for this part. Any two disjoint subsets of S are acceptable.
 $G = \{(1,3), (1,4), (1,5), (2,3)\}$
 $H = \{(2,5), (3,4), (3,5), (4,5)\}$
- $S = \{6, 10, 11, 15, 20\}$
 - Not equally likely since the probability of getting 20 cents is $\frac{C(2,2)}{C(6,2)} = \frac{1}{15}$ and the probability of getting 10 cents is $\frac{C(3,2)}{C(6,2)} = \frac{1}{5}$. Since these are not the same, the sample is not equally likely. Note that we used concepts in section 7.4 to compute these probabilities.
- Answers will vary.
 $E = \{HHH\}$
 $F = \{HHT, HTT, TTT\}$
- Let w = white ball, g =green ball, and y = yellow ball.
 - Note that order is important.
 $S = \{ww, wg, wy, gw, gg, gy, yw, yg\}$
 - $G = \{wg, gw, gy, yg\}$
 - Answers will vary, but pick E such that $E \cap G = \emptyset$
 One answer is: $E = \{ww, wy\}$.
- $S = \{R, E, P, S, N, T, A, I, V\}$
 - $2^{n(S)} = 2^9 = 512$
 - $E = \{E, A, I\}$
- No. There are more red balls in the bag, so the drawing a red ball is more likely than drawing a white ball.
 - See part (a) for the answer since uniform and equally likely mean the same thing.
- $0.2 = 1 - (.15 + .25 + .4)$

(b) $0.4 = .15 + .25$

- Since $P(a) + P(b) + P(c) = 1$ and $P(a) + P(b) = 0.75$, then $P(c) = 0.25$. Similarly $P(a) = 0.55$ and $P(b) = 0.2$.
- $J^C = \{d, e\}$ which means that $P(d) + P(e) = 0.4$ and thus $P(d) = 0.25$. Since all probability adds up to 1 we get that $P(c) = 0.2$
- $J^C = \{a, d, e\}$ which means that
 $P(a) + P(d) + P(e) = 0.45$
 $P(a) + 0.2 + 0.1 = 0.45$
 $P(a) = 0.15$
 Since a and b are equally likely, then $P(b) = 0.15$. Since all probability adds up to 1, we get that $P(c) = 0.4$
- $\frac{20+7}{90} = \frac{27}{90}$
 - $\frac{21}{90}$
- $\frac{25+30}{210} = \frac{55}{210}$
- $\frac{6}{11}$
 - $\frac{6+2}{11} = \frac{8}{11}$
- $\frac{41}{713}$
 - $\frac{55+41+52}{713} = \frac{181-33}{713} = \frac{148}{713}$
 - $\frac{171+199-41}{713} = \frac{329}{713}$
 - $\frac{199+141}{713} = \frac{340}{713}$
- $\frac{85+35}{300}$
 - $\frac{85}{300}$
 - $\frac{58}{300}$
 - $\frac{170+26+154-12-138}{300} = \frac{200}{300}$
- $\frac{30+20+10+10}{1000} = \frac{70}{1000}$
 - $\frac{90+290-30}{1000} = \frac{350}{1000}$
 - $\frac{250+320+260}{1000} = \frac{830}{1000}$
- Use the information to fill in a venn diagram to answer part b and c.

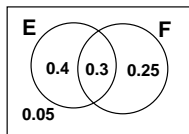


- $0.4 + 0.2 = 0.6$ or $1 - 0.4 = 1 - P(E^C)$
 - 0.4
 - 0.8
- Use the information to fill in a venn diagram to answer part c.



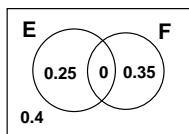
- (a) $0.4 = 1 - 0.6 = 1 - P(E^C)$
- (b) $0.1 = 0.4 + 0.5 - 0.8 = P(E) + P(F) - P(E \cup F)$
- (c) 0.3

20. Use the information to fill in a venn diagram to answer part b and c.



- (a) $0.55 = 1 - P(F^C)$
- (b) 0.3
- (c) $0.3 + 0.25 + 0.05 = 0.6$

21. $P(E \cap F) = 0$, since E and F are mutually exclusive. Use the information to fill in a venn diagram



- (a) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
 $P(E \cup F) = 0.25 + 0.35 - 0 = 0.6$
- (b) 0.65

- 22. (a) $\frac{1}{6} + \frac{1}{8} + \frac{1}{8}$
- (b) $\frac{1}{3} + \frac{1}{6}$
- (c) $1 - (\frac{1}{3} + \frac{1}{6})$
- (d) 2^6 . An event is the same as a subset.
- (e) A and B are mutually exclusive
 C and D are mutually exclusive
- 23. (a) X = a 4 on either die and Y = sum of 5.

		Red Die					
		1	2	3	4	5	6
Green Die	1				X		
	2			Y	X		
	3		Y		X		
	4	X	Y	X	X	X	X
	5				X		
	6				X		

Answer: $\frac{2}{36}$

- (b) X = a 3 on either die and Y = sum of 4.

		Red Die					
		1	2	3	4	5	6
Green Die	1			XY			
	2		Y	X			
	3	XY	X	X	X	X	X
	4			X			
	5			X			
	6			X			

Answer: $\frac{12}{36}$

- (c) X = a 6 on red die and Y = number less than 3 on the green.

		Red Die					
		1	2	3	4	5	6
Green Die	1	Y	Y	Y	Y	Y	XY
	2	Y	Y	Y	Y	Y	XY
	3						X
	4						X
	5						X
	6						X

Answer: $\frac{2}{36}$

24. X = a 4 on either die and Y = sum of 7

		1	2	3	4	5	6
Green Die	1				X		Y
	2				X		Y
	3				XY		
	4	X	X	X	X	X	X

Answer: $\frac{11}{24}$

25. Use a venn diagram to organize the information.

Answer: $\frac{180+85}{500} = \frac{265}{500} = 0.53$

26. $\frac{4*3*4*3}{8*7*6*5}$

- 27. (a) $\frac{8*7*13}{15*14*13}$
- (b) $\frac{8*7*3}{15*14*13}$

28. First count the number of ways to hang the posters on the wall so that the posters of the same type are together. The $3!$ counts the rearrangement of the groups. Divide by the total number of ways to hang the posters on the wall.

Answer: $\frac{(5!4!2!)*3!}{11!}$

29. Choose both Bob and Phill, $C(2,2)$, and choose then 3 of the remaining 18 men. Choose Sara then choose 4 of the remaining 29 women. Divide by the total number of ways to choose 5 men and 5 women. Note: $C(2,2)$ and $C(1,1)$ are both equal to 1 and thus do not have to be included in the answer.

Answer: $\frac{C(2,2)*C(18,3)*C(1,1)*C(29,4)}{C(20,5)*C(30,5)}$

30. Select 3 of the 7 friends and then select 7 of the 93 other applicants.

Answer: $\frac{C(7,3)*C(93,7)}{C(100,10)} = 0.01915$

31. First figure out how many ways the couples may be in the row and then divide by the number of ways 8 people can be placed in a row.

Answer: $\frac{8*1*6*1*4*1*2*1}{8!} = 0.0095$

32. $\frac{C(4,2)+C(5,2)}{C(9,2)}$

33. Use the union formula for counting or probability.
 $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
 where A is exactly 4 green marbles and B is exactly 2 blue marbles.

Answer: $\frac{C(8,4)C(16,2)+C(6,2)C(18,4)-C(8,4)C(6,2)}{C(24,6)}$

34. The 4! is the ordering of the roses on a single shelf. The 8! is the ordering of the flowers on the other two shelves. Multiply by 3 since the roses can be on any of the three shelves.

Answer: $\frac{3(4! \cdot 8!)}{12!}$

35. At least 2 freshmen are the cases: (exactly 2 fr and 1 other) or (exactly 3 fr).

Answer: $\frac{C(12,2) \cdot C(10,1) + C(12,3)C(10,0)}{C(22,3)} = 0.5714$

36. there are 2 defective and 8 good typewriters.

(a) $\frac{C(2,0) \cdot C(8,4)}{C(10,4)} = \frac{70}{210} = \frac{1}{3}$

(b) $\frac{C(2,1) \cdot C(8,3)}{C(10,4)} = \frac{112}{210} = \frac{8}{15}$

37. Select artist A and B and then select two more artist from the remaining 6. Note: $C(2,2) = 1$ so it does not need to be included in the answer.

Answer: $\frac{C(2,2) \cdot C(6,2)}{C(8,4)}$

38. Select 5 banks from the 6 with discounts and then select 1 bank from the 4 without discounts.

Answer: $\frac{C(6,5) \cdot C(4,1)}{C(10,6)} = 0.1143$

39. $\frac{C(13,9)C(12,1)+C(13,10)}{C(25,10)}$

40. For this problem it is easier to calculate total - what you don't want. You don't want less than two born in July. If 0 were born in July then there are 11 other months in which people can be born, 11^7 ways. Next if exactly 1 was born in July there are $1 \cdot 11^6$ ways and we multiply this by 7 so that any of the 7 people could be born in July.

Answer: $1 - \left(\frac{11^7}{12^7} + \frac{7 \cdot (1 \cdot 11^6)}{12^7} \right)$

41. The numerator is a permutation since the day a person is born is acting like a label.

(a) $\frac{P(365,20)}{365^{20}}$

(b) easiest way to count this it to do 1 minus what you don't want, which happens to be part (a).

Answer: $1 - \frac{P(365,20)}{365^{20}} = 0.4114$

42. When picking three people we have the following cases:

Male	Female
0	3
1	2
2	1
3	0

At most two males are the top three cases. notice that we do not want the last case.

Answer: $1 - \frac{C(7,3)}{C(12,3)}$

or $\frac{C(7,2) \cdot C(5,1) + C(7,1) \cdot C(5,2) + C(7,0) \cdot C(5,3)}{C(12,3)}$

43. (a) $P(N|M) = \frac{P(N \cap M)}{P(M)} = \frac{0.25}{.4+.25} = \frac{0.25}{0.65}$

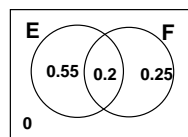
(b) $P(M|N) = \frac{P(M \cap N)}{P(N)} = \frac{0.25}{.15+.25} = \frac{0.25}{0.4}$

44. (a) $P(J|K) = \frac{P(J \cap K)}{P(K)} = \frac{.3}{.3+.22+.09} = \frac{.3}{.61}$

(b) $P(M|K^C) = \frac{P(M \cap K^C)}{P(K^C)} = \frac{.14}{.15+.14+.1} = \frac{.14}{.39}$

(c) $P(M|N) = \frac{P(M \cap N)}{P(N)} = \frac{0}{.09+.1} = 0$

45. Let E = solve the first problem and F = solve the second problem. Fill in a venn diagram with the given information.



(a) $P(F|E) = \frac{P(F \cap E)}{P(E)} = \frac{.2}{.75}$

(b) $P(E^C|F) = \frac{P(E^C \cap F)}{P(F)} = \frac{.25}{.45}$

46. First organize the information into a table.

	Fresh.(F)	Soph.(S)	Total
Male(M)	6	18	24
Female(F)	1	17	28
Total	7	36	42

Answer: $P(S|M) = \frac{18}{24}$

47. (a) $P(O|\text{only rifle}) = \frac{5}{26}$

(b) $P(O|\text{own handgun}) = \frac{58+25}{120} = \frac{83}{120}$

(c) $P(F|\text{own rifle}) = \frac{12+5}{26+35} = \frac{17}{61}$

48. (a) $P(2\text{cds}|\text{over } 25) = \frac{40}{210}$

(b) $P(19 - -25|\text{lessthan } 2\text{cds}) = \frac{70+110}{570} = \frac{180}{570}$

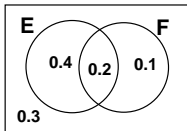
49. We already know two juniors are attending, so we need to determine the remaining three students. We need exactly 1 junior of the remaining 10 and 2 of the 7 remaining students.

Answer: $\frac{C(10,1) \cdot C(7,2)}{C(17,3)}$

50. We already know three questions: 2 difficult and 1 easy, so we need to determine the remaining questions. We need exactly 2 difficult questions of the remaining 6 and 5 easy questions of the remaining 11.

Answer: $\frac{C(6,2) \cdot C(11,5)}{C(17,7)}$

51. Use a venn diagram to organize the information.



- (a) $\frac{P(F^C \cap E)}{P(E)} = \frac{0.4}{0.6}$
- (b) $\frac{P(E^C \cap F^C)}{P(F^C)} = \frac{0.3}{0.7}$
- (c) $\frac{P(F \cap E^C)}{P(E^C)} = \frac{0.1}{0.4}$

52. (a) $P(A^C|C) = \frac{P(A^C \cap C)}{P(C)}$
 $A^C \cap C = \{s_3, s_5\}$
 Answer: $\frac{1/3+1/6}{1/8+1/3+1/6}$

(b) $P(C|B) = \frac{P(C \cap B)}{P(B)}$
 $C \cap B = \{s_3, s_5\}$
 Answer: $\frac{1/3+1/6}{1/3+1/6+1/12}$

- 53. (a) $0.6 * 0.3 + 0.4 * 0.2 = 0.26$
- (b) $0.4 * 0.2 + 0.4 * 0.5 = 0.28$
- (c) $0.6 * 0.7 + 0.4 * 0.3 = 0.54$
- (d) $0.4 * 0.5 = 0.2$
- (e) $P(A \cup G) = P(A) + P(G) - P(A \cap G)$
 $P(A \cup G) = 0.6 + 0.54 - 0.42 = 0.72$
- (f) 0.5
- (g) 0.7
- (h) $P(A|G) = \frac{P(A \cap G)}{P(G)} = \frac{0.6 * 0.7}{0.6 * 0.7 + 0.4 * 0.3}$
- (i) $P(B|R) = \frac{B \cap R}{P(R)} = \frac{0.4 * 0.5}{0.4 * 0.5} = 1$
- (j) $P(A|Y) = \frac{A \cap Y}{P(Y)} = \frac{0.6 * 0.3}{0.6 * 0.3 + 0.4 * 0.2}$
- (k) $P(A|R) = \frac{A \cap R}{P(R)} = \frac{0}{0.4 * 0.5} = 0$

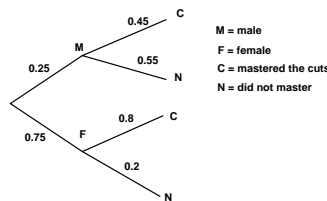
- 54. (a) $0.1 * 0.2 + 0.6 * 0.7 = 0.44$
- (b) $0.3 * 0.25 = 0.075$
- (c) 0.8
- (d) $P(C|G) = \frac{C \cap G}{P(G)} = \frac{0.3 * 0.75}{0.6 * 0.3 + 0.3 * 0.75}$

(e) $P(C) = 0.3$
 $P(E) = 0.1 * 0.2 + 0.6 * 0.7 = 0.44$
 $P(E \cap C) = 0$
 Since $P(E \cap C) \neq P(E) * P(C)$ they are not independent.

- (f) Yes since $P(E \cap C) = 0$
- (g) $P(B) = 0.6$
 $P(E) = 0.1 * 0.2 + 0.6 * 0.7 = 0.44$
 $P(E \cap B) = 0.6 * 0.7 = 0.42$
 $P(E) * P(B) = 0.6 * 0.44 = 0.264$
 Since $P(E \cap B) \neq P(E) * P(B)$ they are not independent.

(h) No since $P(E \cap B) \neq 0$

55. Draw a tree.

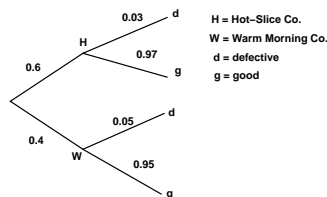


- (a) $P(M|C) = \frac{P(M \cap C)}{P(C)} = \frac{0.25 * 0.45}{0.25 * 0.45 + 0.75 * 0.8}$
- (b) $P(F \cup C) = .75 + .25 * .45 = 0.8625$

56. The third child has a good squirt gun so there are only 59 good guns remaining. Thus the second child could pick any of the 20 bad squirt guns out of the total of 59+20=79 squirt guns.

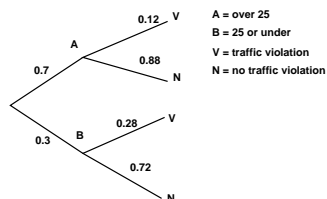
Answer: $\frac{20}{79}$

57. Draw a tree.



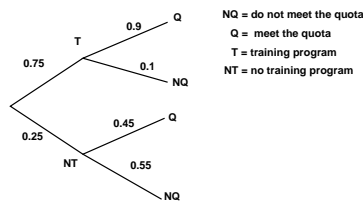
$P(d) = 0.6 * 0.03 + 0.4 * 0.05 = .962$
 Answer: 96.2%

58. Draw a tree.



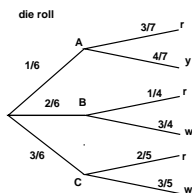
- (a) $P(A|V) = \frac{P(A \cap V)}{P(V)} = \frac{0.7 * 0.12}{0.7 * 0.12 + 0.3 * 0.28}$
- (b) $P(V) = 0.7 * 0.12 + 0.3 * 0.28 = 0.168$

59. Draw a tree.



- (a) $P(T|Q) = \frac{.75 * .9}{.75 * .9 + .25 * .45}$
- (b) $P(NQ \cap NT) = .25 * .55$

60. Draw a tree.



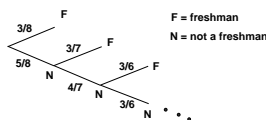
(a) $P(C \cap W) = \frac{3}{6} * \frac{3}{5}$
 (b) $P(B|r) = \frac{2/6 * 1/4}{1/6 * 3/7 + 2/6 * 1/4 + 3/6 * 2/5}$

61. A club and a diamond have been accounted for so there are still 13 hearts remaining and a total of 50 cards remaining.

Answer: $\frac{13}{50}$

62. (a) $\frac{12}{46}$
 (b) $\frac{3}{46}$
 (c) The seventh card was the king of hearts.
 Answer: 0

63. Think of a tree.



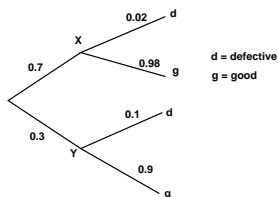
you want $P(N \cap F)$.

Answer: $\frac{5}{8} * \frac{3}{7} = \frac{15}{56}$

64. Draw a tree similar to the one from problem 63

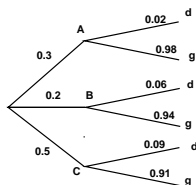
- (a) $\frac{4}{9} * \frac{3}{8} * \frac{5}{7} = \frac{60}{504}$
 (b) By the fifth draw you have to have drawn a green ball. since you stop when you draw a green ball, you will never have a sixth draw.
 Answer: 0

65. Draw a tree.



- (a) $P(g) = 0.7 * 0.98 + 0.3 * 0.9 = 0.956$
 (b) $P(Y|d) = \frac{0.3 * 0.1}{0.7 * 0.02 + 0.3 * 0.1}$
 (c) $P(d \cap Y) = 0.1 * 0.3$

66. Draw a tree.



- (a) $P(g \cap (B \cup C))P(g \cap B) + P(g \cap C) = 0.2 * 0.94 + 0.5 * 0.91 = 0.643$
 (b) $P(g|C) = \frac{P(g \cap C)}{P(C)} = \frac{0.5 * 0.91}{0.5} = 0.91$
 (c) $P(A|d) = \frac{P(A \cap d)}{P(d)} = \frac{0.3 * 0.02}{0.3 * 0.02 + 0.2 * 0.06 + 0.5 * 0.09} = 0.095238$

67. (a) X = 3 or 4 on six sided die
 Y = sum greater than 5.

	1	2	3	4	5	6
1			X	X	Y	Y
2			X	XY	Y	Y
3			XY	XY	Y	Y
4		Y	XY	XY	Y	Y

$P(X|Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{5/24}{14/24}$

Answer: $\frac{5}{14}$

(b) X = odd sum greater than 6
 Y = 4 on either die

	1	2	3	4	5	6
1				Y	X	
2				YX		
3			XY	XY	X	
4	Y	Y	XY	XY	Y	Y

$P(X|Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{3/24}{9/24}$

Answer: $\frac{3}{9}$

(c) X = sum of 4
 Y = sum at most 6

	1	2	3	4	5	6
1	Y	Y	XY	Y	Y	
2	Y	XY	Y	Y		
3	XY	Y	Y			
4	Y	Y				

$P(X|Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{3/24}{14/24}$

Answer: $\frac{3}{14}$

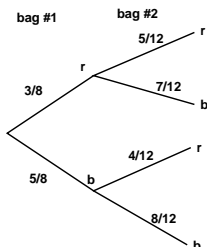
(d) X = sum of 4
 Y = roll was a double

	1	2	3	4	5	6
1	Y		X			
2		XY				
3	X		Y			
4				Y		

$P(X|Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{1/24}{4/24}$

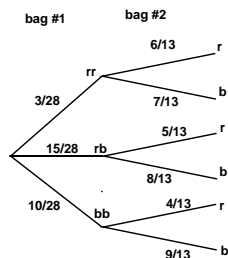
Answer: $\frac{1}{4}$

68. (a) probability tree.



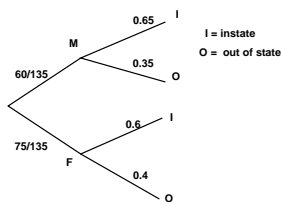
- (b) $\frac{3}{8} * \frac{7}{12} + \frac{5}{8} * \frac{8}{12} = \frac{61}{96}$.
- (c) $P(2^{nd} r | 1^{st} b) = \frac{4}{12}$
- (d) $P(1^{st} r | 2^{nd} b) = \frac{21}{61}$
- (e) $P(1^{st} r | 2^{nd} 4) = \frac{3}{7}$

69. (a) The probability of the first level of the tree was computed using combinations and then converting the answers to fractions.



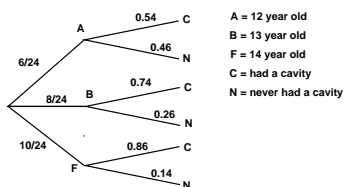
- (b) $\frac{15}{28} * \frac{8}{13} + \frac{10}{28} * \frac{4}{13} = \frac{40}{91}$
- (c) $P(rr \cap b) = \frac{3}{28} * \frac{7}{13} = \frac{3}{52}$
- (d) $P(r|rb) = \frac{5}{13}$
- (e) $P(rb|r) = \frac{75}{133}$
- (f) $P((rr \cup rb)|r) = \frac{(3/28)*(6/13)+(15/28)*(5/13)}{(3/28)*(6/13)+(15/28)*(5/13)+(10/28)*(4/13)} = \frac{93}{133}$

70. Draw a tree.



- (a) $P(I|M) = 0.65$
- (b) $P(F|O) = \frac{.4*75/135}{.35*60/135+.4*75/135} = \frac{10}{17}$
- (c) $P(F) = \frac{75}{135} = \frac{5}{9}$
 $P(O) = \frac{17}{45}$
 $P(F \cap O) = \frac{2}{9}$
 Since $P(F)P(O) = \frac{17}{81}$ is not equal to $P(F \cap O)$ these events are dependent. (i.e. not independent)

71. Draw a tree.

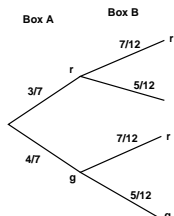


- (a) $P(F|N) = \frac{.14*10/24}{.46*6/24+.26*8/24+.14*10/24} = \frac{35}{156}$
- (b) $P(B) = \frac{8}{24}$
 $P(C) = \frac{6}{24} * 0.54 + \frac{8}{24} * 0.74 + \frac{10}{24} * 0.86 = 0.74$
 $P(B \cap C) = \frac{8}{24} * 0.74 = \frac{37}{150}$
 $P(B) * P(C) = \frac{8}{24} * 0.74 = \frac{37}{150}$

Yes, since $P(B \cap C) = P(B) * P(C)$.

- 72. (a) Since E and F are independent then
 $P(E \cap F) = P(E) * P(F)$
 $P(E \cap F) = 0.6 * 0.3 = 0.18$
- (b) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
 $P(E \cup F) = 0.6 + 0.3 - 0.18$
 Answer: 0.72

73. Since you are drawing an item from each box you can draw this tree to represent the problem.



Answer: $\frac{3}{7} * \frac{7}{13} + \frac{4}{7} * \frac{5}{13}$

74. Note the machines working or not working are independent.

- (a) $(A \text{ breaks down}) * (B \text{ works all day}) + (A \text{ works all day}) * (B \text{ breaks down})$
 Answer: $0.02 * 0.97 + 0.98 * 0.03$
- (b) $(A \text{ works all day}) * (B \text{ works all day})$
 Answer: $0.98 * 0.97$

- 75. $P(E) = \frac{2}{4}$
 $P(F) = \frac{2}{4}$
 $P(E \cap F) = \frac{1}{4}$
 Since $P(E) * P(F) = \frac{2}{4} * \frac{2}{4} = \frac{1}{4} = P(E \cap F)$, these events are independent.

76. Similar to problem 75

Answer: Independent.

77. Similar to Problem 73

$0.075 * 0.87 + 0.925 * 0.13$

- 78. (a) $\frac{9}{10} * \frac{17}{20} * \frac{7}{15}$
- (b) $\frac{1}{10} * \frac{17}{20} * \frac{7}{15} + \frac{9}{10} * \frac{3}{20} * \frac{7}{15} + \frac{9}{10} * \frac{17}{20} * \frac{8}{15}$