Week in Review #2

1. First get rid of the fractions.
   \[ 4 \times \left( \frac{5x}{3} + \frac{y}{4} = 1 \right) \]
   \[ 6 \times \left( \frac{x}{2} - \frac{3y}{5} = \frac{5}{6} \right) \]
   \[ 5x + y = 4 \]
   \[ 2x - 3y = 5 \]
   solve using any methods taught in class.
   Answer: \( x = 1, y = -1 \)

2. Solve both of these lines for \( y \).
   \[ y = \frac{2}{3}x + 3 \]
   \[ y = \frac{k}{2}x + 3 \]
   Both line have the same y-intercept, this means either both lines intersect at infinitely many points (i.e. same slope) or at only one point (i.e. different slopes).
   The slopes are the same when \( \frac{2}{3} = \frac{k}{2} \) or \( k = \frac{4}{3} \). Thus the lines will have exactly one solution whenever \( k \neq \frac{4}{3} \)

3. \( x \) = the number of senior tickets sold.
   \( y \) = the number of adult tickets sold.
   \( z \) = the number of children tickets sold.
   \[ x + y + z = 700 \]
   \[ 6x + 8y + 3.5z = 3512.5 \]
   \[ 3y = z \]

4. \( x \) = the number of Boeing 747s bought.
   \( y \) = the number of Boeing 777s bought.
   \( z \) = the number of Airbus A321s bought.
   \[ x + y + z = 11 \]
   \[ 400x + 300y + 200z = 3200 \]
   \[ 200x + 160y + 60z = 1540 \]

5. \( x \) = the amount invested in low-risk stocks.
   \( y \) = the amount invested in high-risk stocks.
   \( z \) = the amount invested in bonds.
   \[ x + y + z = 82000 \]
   \[ y = x + z \]
   \[ 0.08x + 0.15y + 0.04z = 9050 \]

6. (a) \[ 3d_{2,2} + 2c_{2,1} = 3(5) + 2(-2) = 11 \]
   (b) \[ \begin{bmatrix} 21 & 6 & 12 \\ 18 & 15 & 0 \end{bmatrix} \]
   (c) \[ \begin{bmatrix} 1 & -2 & 2 \\ 3 & 5 & 0 \end{bmatrix} \]

7. simplify the left and right side.
   \[ \begin{bmatrix} 19 & 8x - 3y \\ 4y - 18 & 10 \end{bmatrix} = \begin{bmatrix} 19 & -28 \\ x & 10 \end{bmatrix} \]
   Now solve
   \[ 8x - 3y = -28 \]
   \[ 4y - 18 = x \]
   Answer: \( x = -2, y = 4 \)

8. (a) \[ \begin{bmatrix} 11 & 31 \\ -4 & 43 \end{bmatrix} \]
   (b) not possible since B has 3 columns and D has only two rows.
   (c) \[ \begin{bmatrix} 9x - 1 & 3x + 2 & 8 \\ 13 & 16 & 40 \end{bmatrix} \]
   (d) \[ \begin{bmatrix} x^2 + 2 & x + 5 \\ 2x + 10 & 27 \end{bmatrix} \]
   (e) \[ \begin{bmatrix} 2 & 8 & 16 \\ 4 & 6 & 0 \\ 0 & 2 & 10 \end{bmatrix} \]

9. (a) The numbers in the matrix LM do not represent any usable information.

    The first number in the matrix LM is found by the computation \( 9 \times 30 + 4 \times 7 \).
    The 9 is the number of ounces of Food I and 30 is the number of units of vitamin A in each ounce of Food I giving a result of 270 which is the number of units of Vitamin A eaten for lunch.
    The 4 is the number of ounces of Food II and the 7 is the number of units of vitamin C in each ounce of Food I giving a result of 28 which has no meaning whatsoever.

    (b) The numbers in \( MB^T \) are the number of units of Vitamin A (330) and Vitamin C (125) eaten at breakfast.

10. \[ \begin{bmatrix} 0.4 & -0.2 \\ -1 & 1 \end{bmatrix} \]
11. \[
\begin{bmatrix}
3 & -1 & -1 \\
-4 & 2 & 1 \\
-1 & 0 & 1
\end{bmatrix}
\]

12. no inverse exists.

13. (a) \[
\begin{bmatrix}
2 & 0 & 1 \\
2 & 1 & -1 \\
3 & 1 & -1
\end{bmatrix}
\]

(b) \[
\begin{bmatrix}
2 & 0 & 1 \\
2 & 1 & -1 \\
3 & 1 & -1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} =
\begin{bmatrix}
2 \\
1 \\
4
\end{bmatrix}
\]

(c) \[
A = \begin{bmatrix}
2 & 0 & 1 \\
2 & 1 & -1 \\
3 & 1 & -1
\end{bmatrix},
B = \begin{bmatrix}
2 \\
1 \\
4
\end{bmatrix}
\]

\[
X = A^{-1}B = \begin{bmatrix}
3 \\
-9 \\
-4
\end{bmatrix}
\]

Answer: \(x = 3, y = -9,\) and \(z = -4\)

14. (a) \(X = (B + C)^{-1} \ast E\)

(b) \(X = K \ast (J + A)^{-1}\)