Section 4.5

1. A curve passes through the point (2, 10) and has the property that the slope of the curve at every point P is three times the y-coordinate of P. Find the equation of this curve.

2. A bacteria culture starts with 800 bacteria and will have 1000 bacteria after 30 minutes. Assume that the culture grows at a rate proportional to the number of bacteria present.

   (a) Find a formula for the number of bacteria after \( t \) minutes.
   (b) Find the number of bacterial after 1 day.
   (c) When will the population reach 3200 bacteria.

3. A chemical has a half-life of 18 days. A sample is obtained and 5 days later there remains 50 grams of the chemical.

   (a) Find a formula that will give the amount of the chemical that remains \( t \) days after the sample is obtained.
   (b) What was the initial amount of the sample of this chemical?
   (c) How long will it take until 70% of sample is gone?
4. A turkey is taken from a 350°F oven into a room with a temperature of 80°F. Fifteen minutes later, the turkey is 250°F. Use Newton’s Law of cooling to solve this problem.

(a) Find a formula that will give the temperature of the turkey at time $t$.
(b) What will the temperature be after 40 minutes?

Section 4.6

$y = \arcsin(x) = \sin^{-1}(x)$ means $\sin y = x$ and $\frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$

$y = \arccos(x) = \cos^{-1}(x)$ means $\cos y = x$ and $0 \leq y \leq \pi$

$y = \arctan(x) = \tan^{-1}(x)$ means $\tan y = x$ and $\frac{-\pi}{2} < y < \frac{\pi}{2}$

5. Find the exact value of the following without the aid of a calculator.

(a) $\sin^{-1}\left(\frac{1}{2}\right)$ =

(b) $\arccos\left(-\frac{\sqrt{2}}{2}\right)$ =

(c) $\arctan(-\sqrt{3})$ =

(d) $\tan(\arctan(1.25))$ =

(e) $\sin(\arcsin\left(\frac{\pi}{2}\right))$ =

(f) $\sin^{-1}(\sin\left(\frac{\pi}{7}\right))$ =

(g) $\sin^{-1}\left(\sin\left(\frac{14\pi}{9}\right)\right)$ =
(h) \[ \arccos \left( \cos \left( \frac{16\pi}{9} \right) \right) = \]

(i) \[ \arctan \left( \tan \left( \frac{17\pi}{10} \right) \right) = \]

(j) \[ \tan \left( \arccos \left( \frac{-5}{13} \right) \right) = \]

(k) \[ \sec \left( \arctan \left( \frac{2}{3} \right) \right) = \]

(l) \[ \sin \left( 2 \arctan \left( \frac{2}{3} \right) \right) = \]

6. Write \( \tan(\cos^{-1} 3x) \) without any trig functions.

**Derivatives of Inverse Trigonometric Functions**

\[
\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2} \quad \frac{d}{dx} \csc^{-1}(x) = \frac{-1}{x\sqrt{x^2-1}}
\]

\[
\frac{d}{dx} \cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}} \quad \frac{d}{dx} \cot^{-1}(x) = \frac{-1}{1+x^2} \quad \frac{d}{dx} \sec^{-1}(x) = \frac{1}{x\sqrt{x^2-1}}
\]

7. Prove the derivative rule for \( y = \tan^{-1}(x) \).
8. Find the derivatives of the following.

(a) \( y = \tan^{-1}(5x) \)

(b) \( y = x^2 \arcsin(x^2) \)

(c) \( y = (\cos^{-1}(7x))^3 \)

Section 4.8

9. \( \lim_{x \to 0} \frac{\sin(x) - x}{x^3} \)

10. \( \lim_{x \to \infty} \frac{\ln(x + e^{3x})}{2x} \)

11. \( \lim_{x \to 0} \frac{e^x + e^{-x}}{x^2} \)
12. \( \lim_{x \to \infty} \left( \frac{x^2}{x-1} - \frac{x^2}{x+5} \right) \)

13. \( \lim_{x \to \frac{\pi}{2}} (2x - \pi) \tan(x) \)

14. \( \lim_{x \to 0^+} x^x \)

15. \( \lim_{x \to 0} (1 - 5x)^{\frac{1}{x}} \)