Section 5.1

Answer these questions for each of the graphs.

(A) On what intervals is \( f \) is increasing? decreasing?
(B) On what intervals is \( f \) concave up? concave down?
(C) At what values of \( x \) does \( f \) have a local maximum or minimum?
(D) At what values of \( x \) does \( f \) have an inflection point?
(E) Assuming that \( f \) is continuous and \( f(0) = 0 \), sketch a graph of \( f \).

1. The graph of the derivative, \( f'(x) \), is shown below.

2. The graph of the derivative, \( f'(x) \), is shown below.

Section 5.2

1. For the following functions, find all critical values.

   (a) \( f(x) = xe^{2x} \)

   (b) \( f(x) = |x^2 - 4x| \)

   (c) \( f(x) = x^{\frac{1}{3}}(8 - x) \)
2. Find the absolute and local extrema for these functions by graphing.

(a) \( f(x) = 1 - x^2, \ -2 \leq x < 1 \)

(b) \( f(x) = \begin{cases} 
    x^2, & \text{if } -1 \leq x < 0 \\
    2 - x^2, & \text{if } 0 \leq x \leq 1
  \end{cases} \)

3. Find the absolute maximum and absolute minimum of the given function on the given interval

(a) \( f(x) = x^3 - 2x^2 + x - 5 \) on \([-1, 3]\)

(b) \( f(x) = x^{5/3} + 5x^{2/3} \) on \([-1, 4]\)

(c) \( f(x) = \frac{1}{(x-1)^2}, \) on \([0, 3]\)

4. Sketch a graph of a function \( f \) satisfying the following conditions.

(a) \( x = 2 \) is a critical number, but \( f \) has no local extrema.

(b) \( f \) is continuous with a local maximum at \( x = 2 \), but \( f \) is not differentiable at \( x = 2 \).

(c) \( f \) is defined on the interval \([1, 5]\) but does not have an absolute maximum.
Section 5.3

5. Find the value of \( c \) in the interval \([1, 4]\) that satisfies the conclusion of the Mean Value Theorem for \( f(x) = x^3 + 5 \).

6. Find the intervals where the function is increasing or decreasing and identify all local extrema.
   a) \( f(x) = xe^{x^2-3x} \)
   
   b) \( f(x) = \frac{x}{(x-1)^2} \)

   c) \( f(x) = x \ln(x) \)

   d) \( f(x) = x \sin x + \cos x \) on \([0, 2\pi]\)
7. Determine the intervals where the given function, \( f(x) \) is concave up or concave down and identify all inflection points:

(a) \( f(x) = 5x^7 - 7x^6 + 10 \)

(b) \( f(x) = x \ln(x - 2) \)

8. Given \( f(3) = 8, f'(3) = 0, f''(3) = 6, \)
\( f(7) = 1, f'(7) = 0, \) and \( f''(7) = -5, \) identify any local extrema of \( f \).

9. Find the values of \( A \) and \( B \) so that the function \( f(x) = Ax^3 - 36x^2 + Bx + 7 \) will have an inflection point at \( x = 3 \) and will have a rate of change of \(-36\) at \( x = 2 \).