Week in Review–Additional Material sections 3.3 and 3.4

Section 3.3: Measures of Spread.

- variance (var) and standard deviation (st. dev.) measure the spread of the data about the mean.
  - var = (st. dev)^2
- Chebychev’s Inequality.
- Population versus Samples
  - A population is the entire group.
    - μ is the mean of a population
    - σ is the standard deviation of a population.
  - A sample is a subgroup of the population. This is usually used when it is difficult to get data from the entire population. This information is then used to estimate the information about the population.
    - x̄ is the mean of the sample
    - sx is the standard deviation of the sample

1. A consumer organization evaluates new cars and reports the number of major defects(X) on each car.

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>cars</td>
<td>40</td>
<td>100</td>
<td>209</td>
<td>223</td>
<td>114</td>
<td>91</td>
<td>28</td>
<td>11</td>
<td>4</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Find the variance and the standard deviation.

2. The entire class of two hundred students is asked, “To the nearest hours, how many hours(X) did you study yesterday?”

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>students</td>
<td>16</td>
<td>28</td>
<td>43</td>
<td>43</td>
<td>26</td>
<td>22</td>
<td>14</td>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>

Find the variance and the standard deviation.

3. A certain type of surgery has a success rate of 84%. Assume that the success of the surgery between different patients is independent. This surgery is to be performed on 500 patients in the next year. Let X be the number of patients for who this surgery is a success.

   (a) What values of X are within 1.2 standard deviations of the mean?
   (b) What values of X are more than .75 standard deviations above the mean?
   (c) Find the probability that the number of people having a successful surgery is within two standard deviations of the mean.

4. A hair dryer has a mean lifespan of 24 months and a standard deviation of 3 months, assuming typical usage. Estimate the probability that one of these hair dryers will last between 20 and 28 months.

5. It has been found that with normal usage, the expected lifetime of a product is 3 years with a standard deviation of 4 months. Estimate the number of items in a shipment of 9000 that will last between 30 months and 42 months.

Section 3.4: The Normal Distribution

- continuous probability distribution.
- probability density function
  - f(x) ≥ 0 for values of x
  - area under the curve is 1
- normal distribution (bell curve)
• standard normal curve, \( Z \)
  • \( \mu = 0, \sigma = 1 \)
• To convert values of any normal curve, \( X \), with mean \( \mu \) and \( \sigma \) to \( Z \)-values use \( z = \frac{x - \mu}{\sigma} \)
• calculator commands
  • normalcdf(left cutoff, right cutoff, \( \mu \), \( \sigma \))
  • invNorm(area, \( \mu \), \( \sigma \))

6. Compute the following
   (a) \( P(0.3 < Z < 1.83) = \)
   (b) \( P(Z < 1.5) = \)
   (c) \( P(Z = 1.25) = \)

7. Find the values of \( A \) and \( B \) for the following.
   (a) \( P(Z < A) = .68 \)
   (b) \( P(-B < Z < B) = .48 \)

8. Suppose \( X \) is a normal random variable with \( \mu = 40 \) and \( \sigma = 8 \).
   (a) \( P(32 < X < 53) = \)
   (b) \( P(X > 45) = \)
   (c) Find the value of \( A \) such that \( P(X > A) = .75 \)

9. The random variable \( X \) is normally distributed with a mean of 83 and a standard deviation of 5. Find the percent of the area that is less than 1.3 standard deviations above the mean.

10. The tread life of a tire is normally distributed with a mean of 40,000 miles and a standard deviation of 2000 miles.
    (a) What is the probability that a tire selected at random will have a tread life of more than 35,000 miles?
    (b) In a group what 800 tires, approximately how many of them will last more than 35,000 miles.
    (c) What is the probability that a tire selected at random will have a tread life between 38,000 miles and 44,000 miles?
(d) If 4 tires are installed in a car and experience even wear, determine the probability that all 4 tires will have a tread life between 38,000 miles and 44,000 miles.

(e) If 4 tires are installed in a car and experience even wear, determine the probability that exactly 3 tires will have a tread life between 38,000 miles and 44,000 miles.

11. It has been found that the amount of cheese on a pizza is normally distributed with a mean of 8 oz of cheese with a standard deviation of 0.5 oz.

(a) What is the probability that a pizza selected at random will have less than 7.2 oz of cheese?

(b) Out of 300 pizzas, approximately how many of them will have less than 7.2 oz of cheese?